3. A star’s physical properties are related:

\[ T_{\text{int}} \propto \frac{M}{R}; \quad R \propto M^{1/2} \]

These taken with the relations \( L \propto R^2 T^4 \) and \( T_{\text{int}} \propto T \) imply

\[ L \propto M^3. \]

4. A star’s lifetime can be estimated by considering the amount of fuel and dividing by the rate at which the fuel is burned:

\[ \tau \propto \frac{M}{L} \propto \frac{M}{M^3} = \frac{1}{M^2}. \]

So, massive stars burn out too quickly for life to ever form on planets orbiting them.

5. Advanced Evolution of Stars: Low-mass stars (\( M < 8M_\odot \)) age into a red giant phase in which their surfaces both expand enormously and also cool, but their cores both shrink and heat. This occurs when the H fuel in the star’s core is depleted. A red giant burns He into C and O. When the He fuel is eventually exhausted, the outer stellar portions expand away (Planetary Nebula) and core cools and dies as a White Dwarf.

High-mass stars (\( M > 8M_\odot \)) age into both red and blue giants as their cores burn heavier elements. After the core is converted into iron, nuclear burning ceases and the star collapses to form a neutron star, a violent event accompanied by a supernova explosion. Supernovae are responsible for producing and ejecting most heavy elements into space, where they are incorporated into new stars, planets, and life.

6. Habitable Zones of Stars: Inner edge of habitable zone is where planets are too hot to retain water (e.g., Venus: runaway greenhouse effect). Outer edge is where planets are too cool for liquid water (e.g., Mars: runaway refrigerator effect). The position and size of habitable zones scale with a star’s luminosity.