Q1: Consider a spring, with spring constant $k$, one end of which is attached to a wall. The spring is initially unstretched, with the unconstrained end of the spring at position $x = 0$. The spring is now compressed so that the unconstrained end moves from $x = 0$ to $x = L$. Find the work done by the spring as it is compressed. Express the work done by the spring in terms of $k$ and $L$. Remember the spring experiences the Hooke’s Law force that changes with $x$.

**SOLUTION:**

The spring force exerted by spring with spring constant $k$ is given as

$$ F(x) = -kx $$

The work done by the spring as it is compressed from $x = 0$ to $x = L$ is given as

$$ W = \int_0^L dW = \int_0^L F(x) \cdot dx = \int_0^L -kx \ dx \cos(0) = -\frac{k}{2}x^2 \bigg|^L_0 = -\frac{k}{2}L^2 $$

Q2: Find the force associated with the potential energy of a spring is given as below

$$ U(x) = \frac{1}{2}kx^2 - Fx + \frac{F^2}{k} $$

Here $F$ is some conservative force. Also, state an additional potential energy of your choice that gives the same force.

**SOLUTION:**

The force due to the potential $U(x)$ is given as

$$ f = -\frac{dU(x)}{dx} = -\frac{d}{dx} \left\{ \frac{1}{2}kx^2 - Fx + \frac{F^2}{k} \right\} = -kx + F $$

There could be infinite number of potential energy functions that could give you the same force. Point to note is that the one you choose should give you the same derivative. For e.g

$$ U(x) = \frac{1}{2}kx^2 - Fx + \text{Constant} $$
1:) What is the work done $dW$ when a force $F(x)$ produces a small displacement $dx$? 
(Keep track of the vectors here and multiply accordingly) 

$$dW = F(x) \cdot dx$$

2:) A force $F(x)$ is applied to an object that results in a displacement from position $x_1$ to $x_2$.

What is the net work done in this case $W$ (write in integral form)?

$$W = \int_{x_1}^{x_2} F(x) \cdot dx$$

3:) How is work done $W$ related to the change in potential energy $\Delta U$ of the system?

$$W = -\Delta U$$

4:) How is the force $F(x)$ related to the potential energy $U(x)$ of the system?

$$F(x) \Delta x = -\Delta U \rightarrow F(x) = -\frac{\Delta U}{\Delta x}$$

5:) If an object is raised from height $h_1$ to $h_2$.

The change in potential energy $\Delta U$ is

$$\Delta U = U_1 - U_2 = mg(h_1 - h_2)$$

The corresponding work done $W$ is

$$W = -\Delta U = mg(h_2 - h_1)$$

6:) What is the force exerted by a spring $F(x)$ with spring constant $k$?

$$F(x) = -kx$$

Note the force exerted by you on the spring $F(x)$ with spring constant $k$ will be

$$F(x) = kx$$

7:) The Gravitational force between the earth of mass $M$ & radius $R$ and an object of mass $m$ is $F = \frac{GMm}{r^2}$.

Then the work done to displace the object from the surface of the earth to outer space is

$$W = \int_{R}^{\infty} \frac{GMm}{r^2} dr = \frac{GMm}{R}$$

We can find acceleration due to gravity $g$ is $9.8m/s^2$ from above.

$$mg = \frac{GMm}{R^2} \rightarrow g = \frac{GM}{R^2} = \frac{6.67300 \times 10^{-11} m^3/kg s^2 \times 6371 \times 10^3 m \times 5.9736 \times 10^{24} kg}{(6371 \times 10^3)^2} \rightarrow g = 9.82 m/s^2$$

8:) Two unlike charges $q$ and $-q$ attract each other with a force $F = -\frac{Kq^2}{r^2}$.

The work done to pull them apart where the influence of the force is negligible

$$W = \int_{r}^{\infty} -\frac{Kq^2}{r^2} dr = -\frac{Kq^2}{r}$$