I. BACKGROUND

In 1913 Neils Bohr wrote a three-part article that described the energy levels of the hydrogen atom and reproduced the frequencies of the radiation given off by the hydrogen atom when the electron falls to a lower energy state. In the second part of the article Bohr described the energy levels for a neutral helium atom. "For a system consisting of two electrons and a nucleus of charge $2e$, we may therefore assume the existence of a series of stationary states in which the electron most lightly bound moves approximately in the same way as the electron in the stationary states of a hydrogen atom." While the Bohr model of the atom has been surpassed by further developments in quantum mechanics, one success of the model was the accurate prediction of the transition energies of atomic systems that can be modeled as one-electron atoms.

Atoms that have one electron excited to a high principal quantum number $n$ are referred to as Rydberg atoms. These atoms can be treated, in some respects, as one-electron atoms. The inner region of the atom has $Z$ protons and $Z - 1$ electrons for a net charge of $+1|e|$. This charge attracts the single electron in the high $n$ principal quantum number state. Such atoms do not occur naturally on earth but are common in certain laboratory and astrophysical environments.

The term recombination line is used to describe the radiation that results from an energy level transition of the outer electron in a Rydberg atom. Recombination lines originate in astrophysical environments where hydrogen and other elements are ionized. Electrons then recombine with the ionized elements to form atoms in highly excited states. These atoms can radiate at various frequencies as the electrons make energy level transitions on their way toward the ground state. Recombination lines are most easily observed in what are known as HII regions. These are regions around very hot stars in which the hydrogen is ionized by the ultraviolet radiation from the star. (According to astrophysical nomenclature ionized hydrogen is indicated as HII. Neutral hydrogen is indicated HI.) Recombination lines can also be observed from planetary nebulae, external galaxies, and dark clouds.

Atomic transitions, including recombination lines, are indicated by the chemical symbol of the element involved, the final $n$ of the electron, and a Greek letter indicating the line’s order. For example H53$\alpha$ indicates the radiation given off when the electron in a hydrogen atom drops from the $n = 54$ state to the $n = 53$ state. C64$\beta$ indicates the $n = 66$ to $n = 64$ transition of the outer electron of a carbon atom.

The frequency of the recombination line, $\nu$, can be calculated using the appropriate $n$ values:

$$\nu = \frac{Z_{\text{eff}}^2 m_e M e^4}{8\hbar^3 \epsilon_0 (m_e + M)} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right),$$

where $m_e$ is the mass of the electron, $M$ is the mass of the rest of the atom, $e$ is the electron charge, $\hbar$ is Planck’s constant, $\epsilon_0$ is the permittivity of free space, $n_f$ is the final $n$ value, and $n_i$ is the initial $n$ value. $Z_{\text{eff}}$ is the effective nuclear charge in units of electronic charge. Due to the screening action of the inner electrons, $Z_{\text{eff}}=1$ for atomic recombination lines. Equation (1) is developed from the equation for the energy levels of one-electron systems and includes the reduced mass effect. The energy level equation can be found in modern physics texts.

Substituting the appropriate values and simplifying, Eq. (1) reduces to

$$\nu = \frac{(3.289 841 \times 10^{15} \text{Hz}) \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right)}{1 + \frac{m_e}{M}}.$$

Table I shows the frequency values for the 53$\alpha$ transition for hydrogen, helium, and carbon. These transitions and others, listed at the end of the article, are available using the Haystack Observatory Radio Telescope. The frequency values for a large number of recombination lines have been tabulated for hydrogen and helium.

In astrophysical situations there is generally a relative velocity between the source and the observer of radiation. This velocity alters the observed frequency of the radiation. The radial component of this velocity can be determined by applying the equation for the Doppler effect. For radio astronomical applications the Doppler velocity can be expressed as

$$V_D = \frac{(\nu_0 - \nu_{\text{obs}})}{\nu_0} c,$$

where $\nu_{\text{obs}}$ is the observed frequency, $\nu_0$ is the calculated rest frequency, and $c$ is the speed of light. The Doppler velocity is used to indicate the radial velocity of the emitting region with respect to the local standard of rest. The local standard of rest is used as the reference velocity for the telescope.
expression for the Doppler velocity can be derived from the equation for the Doppler shift using a binomial expansion for velocities small compared to the speed of light. An observed frequency higher than the calculated frequency indicates the distance between the source and observer is decreasing, and \( V_D \) is negative. Velocities transverse to the line of sight must be determined using other methods.

II. RECOMBINATION LINES IN MODERN PHYSICS

Students in many modern physics courses review the development and implications of the quantum mechanical model of one-electron atoms. The usual examples are hydrogen, singly ionized helium, doubly ionized lithium, and so on. Many times the visible Balmer lines of hydrogen are measured in the modern physics laboratory if they were not measured in general physics. Perhaps the visible spectrum of deuterium is calculated and measured to show the effect of changing \( M \). Observations of recombination lines provide another example of the quantum mechanical model of atoms.

Using the internet it is now possible to make remote observations of various recombination lines using the Haystack Observatory Radio Telescope. This observing capability allows students to develop an observing program and control the 37-m telescope and receivers from their home institutions. The details of such observing can be found by reviewing the Haystack web page at http://www.haystack.mit.edu or by contacting Dr. Preethi Pratap at Haystack Observatory.

Figure 1 shows a spectrum of an HI region, the Kleinman Low region of the Orion nebula acquired by student observers at the University of Minnesota, Morris, using the Haystack Observatory Radio Telescope. The spectrum represents about 2 h of observing time. The \( \text{H}53\alpha \) and \( \text{He}53\alpha \) lines are obvious in the spectrum. The helium line may also contain a contribution from carbon atoms. Due to the proximity of the frequencies of the helium and carbon transitions and the observation of carbon lines at various velocities in lower frequency transitions, a contribution of radiation from carbon atoms may exist in the weaker line. A linear baseline has been removed from the spectra. This baseline correction removes any slope or offset in the baseline due to variations in the response of the telescope’s spectrometer or changes in the power received by the telescope due to atmospheric or electrical variations. The dimensions on the ordinate are flux density. These are the standard radio astronomical dimensions of power per unit area per unit bandwidth. The unit of flux density is the Jansky.

From the center frequency of the Gaussian and the frequency of the transition calculated from Eq. (2), the Doppler velocity of the emitting region can be determined using Eq. (3). The full width at half maximum of the Gaussian, \( \nu_{\text{FWHM}} \), is a result of the turbulent and thermal motion of the emitting material. These physical parameters can be extracted from the larger hydrogen feature and from the smaller feature that is a result of emission from helium and carbon. This feature will be referred to as the (He\(^+\)+C) feature. Since the emitting regions for the two features are probably not co-spatial the fit values are not expected to be exactly the same. The fit values and one sigma uncertainties for the hydrogen and (He\(^+\)+C) features are given in Table I. For the (He\(^+\)+C) feature the He frequency has been used to determine the \( V_D \).

The areas of the Gaussians can be determined by using the peak flux density and the width of the Gaussians to integrate the functions. The ratio of the areas of the (He\(^+\)+C) Gaussian to hydrogen Gaussian can be compared with existing values for this transition. The lower limit of the ratio’s value from the present work, \( 0.18 \pm 0.12 \pm 0.07 \), is comparable to previous determinations of the ratio at this location. This ratio is important in determining the chemical evolution of the galaxy. The He/H ratio, which can be extracted from the (He\(^+\)+C)/H ratio, is of great importance in understanding Big Bang nucleosynthesis.

Other recombination lines available by using the Haystack Observatory Radio Telescope are \( \text{H}52\alpha \), \( \text{H}54\alpha \), \( \text{H}64\alpha \), \( \text{H}65\alpha \), \( \text{H}66\alpha \), and \( \text{H}67\alpha \) and the nearby helium lines.

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6See, for example, Paul A. Tipler and Ralph A. Llewellyn, Modern Physics (Freeman, New York, 1999), 3rd ed., pp. 170–181.


9See Ref. 6, pp. 45–47.


12See Ref. 11, pp. 586–590.