Basic Physic of Colliders

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1 Relativistic Kinematic

A collision of two particles can happen in two ways: as a fixed target or as both beams having kinetic energy. In the case of a particle collider, their differences are shown in table 2.

	Fixed Target	Both Beams Moving
Luminosity	Bigger	Low
Target	Solid, Liquid	Gas
E_{CM}	$= \sqrt{m_1^2 + m_2^2 + 2E_1 m_1}$	$= \sqrt{m_1^2 + m_2^2 + 2E_1 E_2 + 2p_1 p_2}$
	$\sim \sqrt{2E_1 m_2}$	$\sim \sqrt{4E_1 E_2}$
Cost Increases on	$\sqrt{E_{CM}}$	E_{CM}
Momentum	$k \sim \frac{1}{2}E_{CM}$	$k \sim \frac{1}{2}E_{CM}$

Table 1: The difference between colliding particles with a target fixed or both beams moving.

2 Luminosity

The geometric way of defining *luminosity* is

$$\mathcal{L} = \frac{N_A . N_B f}{A} \tag{2.1}$$

$$\boxed{\text{LUMINOSITY } (\mathcal{L}) = \text{INCIDENT BEAM FLUX} \times \text{MEAN TARGET DENSITY}}$$

The product of luminosity times cross section gives the rate of a process, R, as in (2.2). A non-strong interaction with small cross section needs bigger luminosity, which means squeeze as many as particles in the smaller cross section area possible.

$$R = \frac{dN}{dt} = \mathcal{L} \times \sigma. \tag{2.2}$$

The number of events produced is given integrating (2.2) on time:

$$N_e = \sigma \times \int \mathcal{L} dt, \tag{2.3}$$

however, since not all the events are recorded, one needs to take account the efficiency of the trigger to identify the collision of interest, ϵ, therefore (2.3) becomes

$$N_e = \sigma \times \int \mathcal{L} dt \times \epsilon. \tag{2.4}$$

2.1 Example 1: Finding the Integrated Luminosity

Supposing an experiment has $\mathcal{L} = 10^{31} cm^{-2} s^{-1}$, the calculus of the integrated luminosity in a year ($\sim 10^7$ s) is given by

$$\int \text{ dt } \mathcal{L} = 10^{31} c^{-2} s^{-1} . 10^7 s = 10^{14} \text{ barns } = 100 \text{ pb }^{-1}. \tag{2.5}$$

2.2 Example 2: Finding Events Produced

In the integrated luminosity in the last example, 100 pb $^{-1}$, one can calculate the number of $p\bar{p} \to t\bar{t}$ events produced at $\sqrt{s} = 7$ TeV (LHC). This process at this E_{CM} has a cross section of $\sigma \sim 165$ pb, thus one has

$$N_{ev} = \sigma \times \int \mathcal{L} \text{ dt } = 165 \times 100 = 16,500 \text{ pairs.} \tag{2.6}$$

2.3 Example 3: Achieving more Luminosity

The calculus of the size of the beam spot needed to achieve $\mathcal{L} = 10^{34} cm^{-2} s^{-1}$ at a machine frequency 11 kHz and $n_b = 2808$ (LHC), with about $N_b = 10^{11}$ protons per bunch is

$$\sigma_{x,y} = \sqrt{11 \times \frac{2808 \times (10^{11})^2}{4\pi \times 10^{34}}} = 15 \mu m. \tag{2.7}$$

3 Rapidity and Pseudorapidity

Pseudorapidity is the angle of the particle relative to the beam axis, (3.1).

$$\eta = - \text{ ln tg } \frac{\theta}{2}, \tag{3.1}$$

where θ is the angle between p_T and the beam axis. If $\theta = 90^o, \eta = 0$ and $\theta = 0, \eta = \infty$. In the limit of the particle is close to the speed of the light ($m \sim 0$), one can define rapidity as (3.2). In the Forward region, $\eta > 1, \theta \sim 0$, in the Backward region, $\eta < -1, \theta \sim \pi$, and in the central region $\eta = 0, \theta \sim \frac{\pi}{2}$.

$$y = \frac{1}{2}\ \ln\left(\frac{E+p_L}{E-p_L}\right) = \ \tanh\ ^{-1}\frac{p_L}{E}, \tag{3.2}$$

where p_L is the component of the beam in direction of propagation and θ is not a Lorentz invariant. One can then define the *forward direction* as the direction of the detector that is close to the beam axis at high η. Any change of the rapidity is Lorentz-invariant under boosts in the beam-direction. For a $\sim$ massless particle, the rapidity and the pseudorapidity are the same.

4 Transverse Quantities

The transverses quantities are useful because p_z is not conserved.

$$\begin{aligned} P_T &= p\text{sin}\theta, && (4.1)\\ E_T &= E\text{sin}\theta, && (4.2)\\ m_T^2 &= \sqrt{E_T^2 - P_T^2}. && (4.3) \end{aligned}$$

The missing transverse energy, $\not{E}_T$, is defined as

$$\not{E}_T = -\sum_i E_T^i.\hat{n}_i = -\sum E_T. \tag{4.4}$$

5 Cross Section

The *cross section* of a collision is given by:

$$\sigma = \frac{\text{Number of Events}}{N_A.N_B} \times A, \tag{5.1}$$

where A is the cross section area and N_i are the number of particles of each i beam. If the beam collides at the frequency f Hz, at a rate R, the number of elements are given by (2.2).

In natural units $c = \hbar = 1$ the natural unit of σ is barn, where $1b = 10^{-24}$ cm^2= 2568 GeV^{-2}. Roughly speaking one can say that $\sigma_{total} = \frac{1}{E_{CM}^2}$. However for higher energies the σ for a specific process is lower by at least an order of magnitude, e.g $e^+e^- \rightarrow Z$ has a cross section on resonance $\sqrt{s} = M_Z$ given by:

$$\begin{aligned} \sigma_Z &= 40 \text{ nb},\\ \sigma_{geo} &= \frac{\pi}{M_Z} \sim 2500 \text{ nb}. \end{aligned}$$

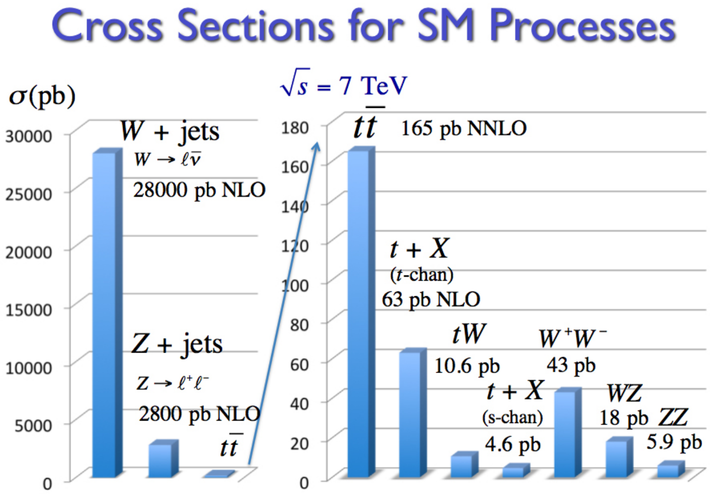

Figure 1: Cross sections for process at LHC.

6 The Invariant Mass

$$M_{NN}^2 = (p_1 + p_2)^2 = (E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2, \tag{6.1}$$

$$= m_1^2 + m_2^2 + 2(E_1E_2 - |\vec{p}_1||\vec{p}_2| \cos\theta) \tag{6.2}$$

In the relativistic limits $E = |\vec{p}|$ and $m_i \ll E_i$, so

$$M_{NN} = 2\sqrt{E_1E_2 \sin\frac{\theta_{12}}{2}} \tag{6.3}$$

7 Particle Spectra

The invariant cross-section is defined as

$$E\frac{d^3\sigma}{dp^3} = \frac{d^3\sigma}{dydp_T^2} = \frac{1}{p_T}\frac{d^3\sigma}{dydp_T}. \tag{7.1}$$

The components of the energy are $E_L = E\ \cos\theta$ and $E_T = E\ \sin\theta$ so

$$\langle E_L \rangle = \frac{E\int_o^{\pi/2}\ \cos\theta\ \sin\theta}{\int\ \sin\theta} = \frac{E}{2} = \frac{\sqrt{s}}{2}, \tag{7.2}$$

$$\langle E_T \rangle = \frac{E\int_o^{\pi/2}\ \sin^2\theta}{\int\ \sin\theta} = \frac{\pi E}{4} = \frac{\pi\sqrt{s}}{4}. \tag{7.3}$$

Therefore, we measure

$$\frac{dE_T}{d\Omega} = \frac{E}{4\pi} = \frac{\sqrt{s}}{4\pi} = \frac{1}{2\pi}\frac{1}{\sin^2\theta_{CM}}\frac{dE_T}{d\eta}, \tag{7.4}$$

where

$$\frac{dE_T}{d\eta} = \sin\theta_{CM}\frac{dE}{d\eta}.$$

8 Typical Apparatus

The beam is called *store* , basically after the luminosity (number of collision per second) drops too low to be useful, the store is ended and the collider prepare a new store. After colliding, the resulting particles pass though the following tracking apparatus:

Tracking Chamber Tracks charged particles giving their momenta, in general made of Silicon. They are also Vertex Detectors, aiming track the trajectories of particles close to the interaction point, primary and secondary vertices.

Solenoid Creates the magnetic field aligned in the direction of the beampipe.

EM and Hadronic Calorimeters It is a dense absorber that identifies the trajectory and energy of EM and hadron particles respectively, by stopping them. In the D0 at Tevatron, the EMCAL is made of Uranium, then the particle interacts heavily, loosing completely its energy.

Muon Chamber High energetic muon are rare and if there are tracking in this chamber, it might represent interesting processes.

Trigger Most of the collisions are *soft* , which means that they do not produce interesting events, the function of the trigger is select those that might be relevant. For example at D0 on Tevatron, there are about 1.7 million of collisions every second, the trigger reduces this number to 100 collisions recorded every second. Possible analyses are events with large amount of missing energy; kinematics of Z boson decaying to a pair of electron (Higgs), etc. Usually, if there is no electrons or muons on the sample, it will be count as a background event.

9 Type of Beams

e^+e^- The target in the case of a electron-positron beam is very small $\sim m_e$, therefore the system has small E_{CM}. However, this is a good system to study various spin-one mesons by e^+e^- resonances, such as $c\bar{c}, b\bar{b}, \tau\bar{\tau}$, since hadrons beams involve six incoming valence quarks. With $E_{CM} = 110$ GeV, LEP produced $e^+e^- \rightarrow W^+W^-$.

$pp, p\bar{p}$ Proton-antiproton beams has higher E_{CM} but the resultant quark's E_{CM} are much lower, in the order of 15-50%. Hadrons colliders have no cyclotron radiation as e^-e^+ does.

ep This kind of beam is good for studying the lepton-quark scattering, such as at HERA, which worked at $E_{CM} = 315$ GeV, effective 140 GeV.

For two particles A and B colliding, in the case of e^+e^-, they are A and B, in the case of pp, they are the *partons*. In either case one neglects the masses when compared to the energy. The four-moment vectors for both means in terms of E_{CM} (showed in table 2) are:

$$\begin{aligned} p_A &= (E, 0, 0, +E), \\ p_B &= (E, 0, 0, -E). \end{aligned}$$

One then has $E_{CM} = 2E$ and in terms of the Mandelstam variable, $s = (p_A + p_B)^2 = E_{CM}^2$ and $E_{CM} = \sqrt{s}$.

10 Methods of Acceleration

From the basic equation for the Lorentz force given by (10.1) is possible to see that one cannot use magnetic fields to accelerate particles, since this force depends initially on the velocity and it is perpendicular to the direction of propagation.

$$F = q(E + v \times B). \tag{10.1}$$

The way particle are accelerated in colliders are by radio frequency (RF) EM-fields, either in circular or linear colliders. In order to reach higher kinetic energies, the particle must travel through more acceleration gaps, and in this sense, the former is more appropriate. A time-variating magnetic field is used to bend the particle around a circle with radius obtained from

(10.1) and given by (10.2). Clearly one can see that the radius is proportional to the momentum and need to be balanced to an inversely proportionally magnetic field.

$$r = \frac{p}{qB}. \tag{10.2}$$

This is the principle of the synchrotrons, which are composed by many bend magnets, deflecting the beam by small angles. It is important to have in mind that from (10.2), a particle with bigger momentum will bend less in a magnetic field, which is obvious since the particle spend less time thought the force of this field.

11 What is Possible to Identify

The identification of the flood of secondary particles resulting from colliders beams are given by the energy relation

$$(E_T)^2 = (pc)^2 + (E_0)^2. \tag{11.1}$$

The possible results are:

High Energy Jet Localized peak of hadron in a small group of adjoint Calorimeter cells, giving the momentum vector of the original particle.

Electrons, Positrons On the charged Tracker and on the Calorimeter.

Hadrons On the charged track and on the Calorimeter.

Muons On the charged Tracker and the Muon Chambers.

Photons On the Calorimeter, as signal exactly as the electrons but without signal on the Tracker. Any number of photons in the same Calorimeter cell give the same signal.

Missing Transverse Energy, Momentum The momentum component of all hadrons, $e^{\pm}, \mu^{\pm}, \gamma$, transverse to the beam axis can be measured. Any significant imbalance in p_T can be attributed to penetrating neutrals which passed unidentified: neutrinos, photinos, etc.

Examples of identification of process (at CMS at LHC) can be seem at figures 2 and 3.

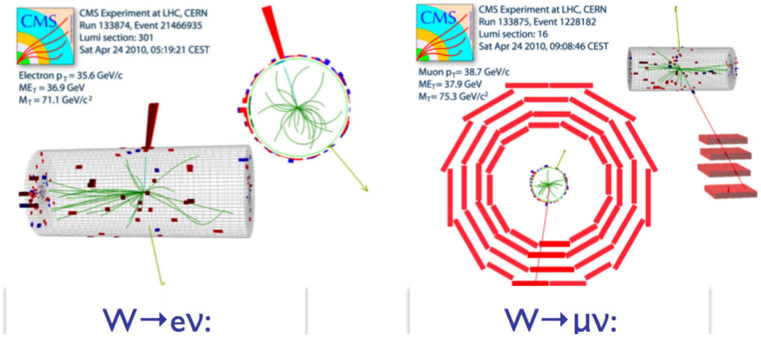

Figure 2: Tracking of $W \to e\nu$ and $W \to \mu\nu$ at CMS in the LHC.

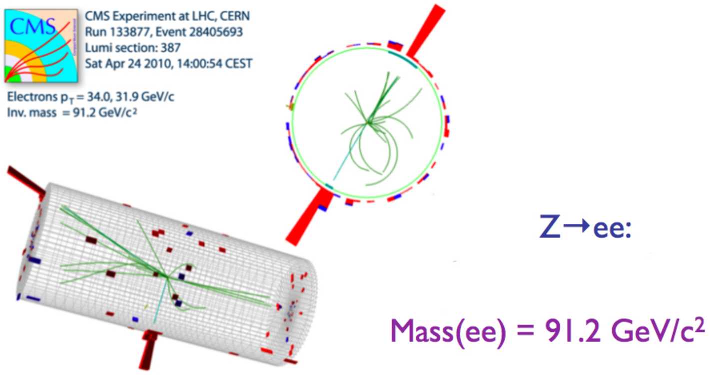

Figure 3: Tracking of $Z \to ee$ at CMS in the LHC.

12 Colliders

12.1 RHIC

The *Relativistic Heavy Ion Collider* is a collider of heavier ions (pp, Aup, $CuCu$, $AuAu$) which aims the study the quark-gluon plasm, which by analyzing photons and leptons one can infer the temperature of the collision and other characteristics. It is the only collider that produce polarized photons. The general purpose experiments are Star (hadrons) and Phoenix (rare, EM particles).

12.2 Tevatron

The *Tevatron* is a circular accelerator, at the Fermi National Accelerator Laboratory, constructed in 1983 and it collides $p\bar{p}$ beams up to 1 TeV each, inside a ring of about 6 km. The quark t, the B_S oscillation, among others were discovered on this collider. The general purpose experiments are CDF and D0.

12.3 LHC

The *Large Hadron Collider*, at CERN, is starting operations colliding beams of pp. The general purpose experiments are ATLAS and CMS, the more specific experiments are ALICE (quark-gluon plasm), TOTEM and LHCb (CP violations).

	LHC	TEVATRON
CM-Energy	14 TeV	1.96 TeV
Bunches	2808	36
Luminosity	$10^{33} - 10^{34} cm^{-2} s^{-1}$	3×10^{32}
Integrated $\mathcal{L}$/Year	10 - 100 fb^{-1}	$>$ fb^{-1}

Table 2: Differences between the Tevatron and the LHC colliders.

References

[1] PESKIN, M. and SCHROEDER, D.; Introduction to quantum field theory.

[2] Particle Data Group, 2010, `http://pdg.ihep.su/2010/figures/figures.html`.