Photonproduction of Heavy Quarks

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Introduction

One of the motivations of collider experiments is to discover new heavy objects, therefore it is important to test the understanding of the predictions for the know heavy objects, such as top, bottom and charm quarks. The description of hadronic production is possible in perturbative QCD when the mass of the produced quark is large when compared to the GeV scale (scale of strong interactions). Following the textbooks [2] and [3], together to the original paper [1] I will evaluate the process of photon-gluon fusion in heavy quark flavor.

1 Photoproduction of Heavy Quarks

We want to calculate the cross section for the photon-gluon fusion process, $\gamma P \to Q\bar{Q}$, figure 1. From the partonic model, the process can be factorized into the partonic subprocess $g\gamma \to Q\bar{Q}$. This process differs from the simple case $g\gamma \to q\bar{q}$ since we need to take into account the terms with mass (the limit $m \to 0$ does not work anymore). Moreover, the difference between a boson in QED, γ, and a boson in QCD, g, is taken into account by including the non-abelian group term and replacing the right coupling constants α_s for QCD and α for QED.

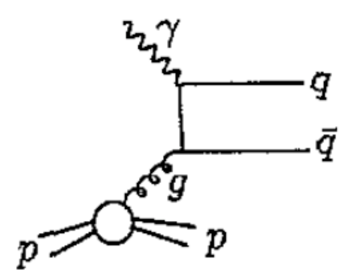

Figure 1: Photoproduction of heavy quarks.

Unlike *deep inelastic scattering*, the process is sensitive to the gluon content of the proton already on leading order (LO). In the massive approach, only three active flavors (u,s,d) are assumed in the proton. For instance, for charm production, the photo-gluon fusion is the dominant process.

2 Factorization Theorem

The hadron is composed of incoherent (free, independent) quarks and gluons. The quarks and gluons undergo the hard scattering and then produces hadrons again. The middle can be treated by perturbation theory. The first part cannot but it is universal: the description of hadrons in terms of quarks and gluons are the same for all process and energies, by the parton distribution functions (PDF). All this is true up to corrections of order Λ_{QCD}/q^2, where q^2 is the invariant characterizing how high is the energy process.

Therefore, perturbative calculations for heavy quark production are performed in the context of the factorization theorem,

$$\sigma_{a\to c} = \mathcal{F}_{a\to b}(x,Q^2)\otimes\hat{\sigma}_{b\to c} + \mathcal{O}(\Lambda^2_{QCD}/Q^2) \qquad (1)$$

Despite the fact that it is not possible to find the structure functions, $\mathcal{F}_{a\to b}(x,Q^2)$, from first principles, we can although find the partonic subprocess, $\hat{\sigma}_{b\to c}$.

3 Relevant Parton Subprocess

The simple two-body scattering processes of quarks, antiquarks and gluons are the elementary process of QCD perturbation theory, in the same way as the two-body QED process. They are the basic hadronic hard-scattering reactions at leading α_s. All the cross sections are of order α_s^2 and the coupling should be evaluated at a typical momentum transfer of the reaction, for example, $Q^2 = \hat{t}$.

Another point in the evaluation is that the gluon external line cannot be replaced the polarization sum with $g^{\mu\nu}$, as in the case of photons. It must be evaluated summing over physical transverse polarization states.

From previous homework, we have found in the massless limit,

$$\frac{d\sigma}{d\hat{t}}(q\bar{q}\to gg) = \frac{32\pi\alpha_s^2}{27\hat{s}^2}\Big[\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} - \frac{9}{4}\Big(\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\Big)\Big]. \qquad (2)$$

By crossing symmetry, we can find $d\sigma(gg\to q\bar{q})$ averaging over the gluons rather than quark colors, this will give a factor of $3^2/8^2$,[1]

$$\frac{d\sigma}{d\hat{t}}(gg\to q\bar{q}) = \frac{\pi\alpha_s^2}{6\hat{s}^2}\Big[\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} - \frac{9}{4}\Big(\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\Big)\Big]. \qquad (3)$$

[1] Remember the non-Abelian formulae tr $[t^a t^a] = C_2(r)\times$ tr$[1] = \frac{4}{3}\times 3$, where 3 is the color sum and therefore from QED to QCD we only make the replacements $g^2 \to \frac{4}{3}g^2$.

For $\frac{d\sigma}{d\hat{t}}(g\gamma\to q\bar{q})$, we make $\alpha_s^2\to\alpha_s\alpha$ and we only have one gluon color average factor, therefore instead of $3^2/8^2$ we multiply by $3^2/8$. Also we remember that the last term of equation 3 was due the triple gluon vertex and we don't need it on our $g\gamma$ process. The differential cross section becomes

$$\frac{d\sigma}{d\hat{t}}(g\gamma\to q\bar{q}) = \frac{4\pi\alpha_s\alpha}{3\hat{s}^2}\Big[\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}}\Big].$$

However, we don't want the massless limit as the previous results anymore. From the complete QED annihilation differential cross section (computed in the first exercise of the exam), we have

$$\frac{d\sigma}{dt}(e^-e^+\to\gamma\gamma) = \frac{-2\pi\alpha^2}{s^2}\Big[\frac{u}{t} + \frac{t}{u} + 4 \quad m^2\Big(\frac{1}{t}+\frac{1}{u}\Big) - 4m^4\Big(\frac{1}{t}+\frac{1}{u}\Big)^2\Big)\Big].$$

Replacing the same QCD factors as before, our partonic subprocess differential cross section for heavy quark production from photon-gluon production becomes

$$\frac{d\sigma}{d\hat{t}}(g\gamma\to Q\bar{Q}) = \frac{4\pi\alpha_s\alpha}{3\hat{s}^2}\Big[\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} + 4m^2\Big(\frac{1}{\hat{t}}+\frac{1}{\hat{u}}\Big) - 4m^4\Big(\frac{1}{\hat{t}}+\frac{1}{\hat{u}}\Big)^2\Big)\Big]. \qquad (4)$$

We should include the charge of the the final quarks, Q_f^2, in this equation/[2].

4 Parton-model Cross Section

The partonic cross section can be combined with the parton distribution functions of gluon and γ to predict the full cross section,

$$\frac{d^3\sigma}{dx_1dx_2d\hat{t}}(1+2+Y\to 3+4+Y) = f_1(x_1)f_2(x_2)\frac{d\sigma}{d\hat{t}}(1+2\to 3+4)$$

[2] For charm production, $m\sim 13$ GeV, $e_c = 2/3$, for top production, $m\sim 172$ GeV, $e_t = 2/3$, for bottom production, $m\sim 42$ GeV, $e_b = -1/3$.

Recalling 4, we have then[3]

$$\sigma(\gamma P \to Q\bar{Q}Y) = \mathcal{F}(x,Q^2) \otimes \hat{\sigma}_{\gamma g \to Q\bar{Q}} + \mathcal{O}(\frac{\Lambda^2_{QCD}}{Q^2}),$$
$$= \int_0^1 dx_1 \int_0^1 dx_2 \mathcal{F}_\gamma(x_1)\mathcal{F}_g(x_2)\sigma(\gamma g \to Q\bar{Q}Y).$$

5 Structure Function for $\gamma g \to q\bar{q}$

The structure functions in function of the scale/renormalization group is calculated in the reference [2],

$$\mathcal{F}^g(x,Q^2) = x\sum_{q\bar{q}} e_q^2 \frac{\alpha_s}{2\pi}\Big(P_{qg}(x)\,\ln\frac{Q^2}{\kappa^2} + C_g(x)\Big), \quad (5)$$

where $P_{qg}(x) = \mathrm{Tr}[x^2 + (1-x)^2]$ is the splitting function. There is a logarithmic singularity from vanishing quark virtuality $\kappa^2 \to 0$. The color factor Tr = 1/2 is obtained from Tr $t^a t^b = \mathrm{Tr}\delta^{ab}$, Tr=1/2, and the sum is over the n_f massless quark and antiquark flavors which contribute.

6 Results from [1]

The original calculation from Fritzsch and Streng, in 1978, gives the following partonic subprocess cross section

$$\hat{\sigma}(\hat{s}) = \frac{\pi\alpha\kappa e_h^2}{\hat{s}}[(3-\beta^4)\ln[(1+\beta)/(1-\beta)] - 2\beta(2-\beta^2)],$$

where e_h was the hadron quark charge, $\beta = (1 - (4m^2/\hat{s}))^{1/2})$, and $\hat{s} = (p_\gamma + p_g)^2$. The paper also claims that at very high energies ($E_\gamma \gg m$) the total cross section for producing a pair of flavored hadron $Q\bar{Q}$ is given in lowest order by the factorization

$$\sigma(\gamma g + Y \to Q\bar{Q} + Y) = \int_{m^2/s}^1 dx G(x)\hat{\sigma}(\hat{s})$$

where $s = 2pM + M^2$, p the energy of incoming photon in laboratory system, and M the nucleon mass.

[3] All final cross sections should have a delta function in the sum of the momenta which I omit here in every result.

References

[1] Fritzsch and Streng, Phys. Lett. 7813 (1978).

[2] Ellis and Stirling and Webber, QCD and Collider Physics.

[3] Peskin and Schroeder, Quantum Field Theory.