The vector meson mass in the large $N$ limit of QCD

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Abstract
The vector meson mass is computed as a function of quark mass in the large $N$ limit of QCD. We use continuum reduction and directly compute the vector meson propagator in momentum space. Quark momentum is inserted using the quenched momentum prescription.

Key words: Large $N$ QCD, Vector meson masses, Low energy constants

Meson masses remain finite in the ’t Hooft limit of large $N$ limit of QCD in four dimensions [1]. Chiral symmetry is broken and the value of the chiral condensate has been measured on the lattice for overlap fermions using random matrix theory techniques [2]. The result can be summarized as [3]

$$\frac{\Sigma(b)}{T_c^3(b)} = 0.828 \left[ \ln \frac{0.268}{T_c(b)} \right]^{\frac{3}{22}}$$

where $b = \frac{1}{g^2 N}$ is the bare ’t Hooft coupling on the lattice. The deconfining temperature, $T_c(b)$, is also known from a lattice calculation [4] and is given by

$$b_I = 11 \left( \frac{48\pi^2 b_I}{11} \right)^{\frac{1}{2}} e^{-24\pi^2 b_I}. \quad (2)$$

Continuum reduction holds if $L > \frac{1}{T_c(b)}$ [4] and meson propagators can be directly computed in Euclidean momentum space without any finite volume effects. The pion mass as a function of quark mass, $m_o$, was computed on the lattice using overlap fermions and the pion decay constant is given by [3]

$$\frac{f_\pi}{\sqrt{NT_c(b)}} = 0.269. \quad (3)$$

In this letter, we present results for the mass of the vector meson, $m_\rho$, as a function of the quark mass, $m_o$, using the same technique as the one used for the computation of the pion mass in [3]. The $\rho$ propagator is computed using

$$\mathcal{M}_{\mu\nu}(p, m_o) = \text{Tr} \left[ S_{\gamma\mu} G(U_{\mu} e^{i p \mu}, m_o) S_{\gamma\nu} G(U_{\mu} e^{-i p \mu}, m_o) \right]. \quad (4)$$

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• $G(U_\mu, m_\sigma)$ is the lattice quark propagator computed using overlap fermions in a gauge field background given by $U_\mu$.

• The phase factors, $e^{\pm 2\pi \mu k}$, multiplying the gauge fields correspond to the force-fed momentum of the two quarks in the quenched momentum prescription.

• The meson momentum was chosen to be

\[ p_\mu = \begin{cases} 0 & \text{if } \mu = 1, 2, 3 \\ \frac{2\pi k}{N_L} & \text{if } \mu = 4. \end{cases} \quad (5) \]

• $S$ smears the operator in the zero momentum directions using the inverse of the gauged Laplacian.

The $\rho$ meson is made up of two different quarks (say $u$ and $d$) with degenerate quark masses. Since the associated vector currents are conserved, the propagator, after averaging over gauge fields, will be of the form

\[ \mathcal{M}_{\mu\nu}(p, m_\sigma) = \frac{A(p_\mu p_\nu - p^2 \delta_{\mu\nu})}{p^2 + m^2_s(m_\sigma)}, \quad (6) \]

assuming the propagator is dominated by the lowest vector meson state. Our numerical result is consistent with the above form. We found all off-diagonal ($\mu \neq \nu$) terms and the $\mu = \nu = 4$ term to be zero within errors for the specific choice of momentum in (5) and we also found the $\mu = \nu = 1, 2, 3$ terms to be the same within errors in our small test runs. Since the evaluation of the quark propagators is the computationally intensive part, we set $\mu = \nu = 1$ and obtained a value for the $\rho$ meson mass at six different quark masses by fitting it to the form in (6).

Four different couplings were used in [3] for the computation of the meson masses. We found two of those couplings to be too strong for the computation of the $\rho$ mass. We report here, the results for the $\rho$ mass at two couplings, namely, $b = 0.355$ and $b = 0.360$. Chiral perturbation theory for vector mesons [5] suggests that we fit the data to the form

\[ M_\rho = \tilde{M}_8 + \Lambda_2 M + \delta M_\rho, \quad (7) \]

where

\[ M_\rho = \frac{m_\rho}{T_c(b)}; \quad M = \frac{m_\sigma \Sigma}{T^2_c(b)}; \quad (8) \]

are the mass of the $\rho$ meson and the renormalization group invariant quark mass\(^1\) measured in units of the deconfining temperature. The two coefficients

\(^1\) $M$ denoted the sum of the two quark masses comprising the $\rho$ meson and hence the factor of 2 in the formula for $M$. 

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in (7) are two of the coefficients in the mass term in the chiral lagrangian, namely,

\[ \mathcal{M}_8 = \frac{\bar{\mu}_8 T_c(b)}{\Sigma_3(b)}, \quad \Lambda_2 = \frac{\lambda_2 T_c^3(b)}{\Sigma}. \]  

(9)

The data is plotted in Fig. 1. The result for the chiral condensate in (1) was obtained such that the pion mass as a function of the quark mass scaled properly in the range of coupling from \( b = 0.345 \) to \( b = 0.360 \). It is not necessary that this eliminates finite lattice spacing effects on all quantities and we do see effects of finite lattice spacing effects in Fig. 1. A linear fit performs quite well at both the couplings to yield consistent estimates for \( \mathcal{M}_8 \) and \( \Lambda_2 \). The results are shown both in Fig. 1 and Table 1.

Chiral perturbation theory suggests that \( \delta M_\rho \) in (7) should lead off as \( M_\rho^2 \) and the coefficient of this leading term should be negative. A fit with a \( M_\rho^2 \) term is shown in Fig. 1 and we see that the coefficient at \( b = 0.360 \) is consistent with it being negative. The error in this coefficient is rather large.

**Figure 1:** A plot of \( \rho \) mass as a function of the pion mass in dimensionless units.

**Table 1:** Simulation parameters, critical box size, bare chiral condensate along with the estimates for the two coefficients \( \mathcal{M}_8 \) and \( \Lambda_2 \) in the mass term of the chiral lagrangian.

<table>
<thead>
<tr>
<th>( L )</th>
<th>( N )</th>
<th>( b )</th>
<th>( L_c(b) )</th>
<th>( \Sigma_3(b) )</th>
<th>( \mathcal{M}_8 )</th>
<th>( \Lambda_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>19</td>
<td>0.355</td>
<td>6.96</td>
<td>0.1265</td>
<td>5.39(54)</td>
<td>1.51(32)</td>
</tr>
<tr>
<td>11</td>
<td>17</td>
<td>0.360</td>
<td>8.01</td>
<td>0.1130</td>
<td>5.87(37)</td>
<td>1.76(20)</td>
</tr>
</tbody>
</table>
Using the result for $\bar{M}_8$ in Table 1 for $b = 0.360$ and the result for $f_\pi$ in (3), we have
\begin{equation}
\bar{\mu}_8 = \frac{21.8 \pm 1.4}{\sqrt{N}} f_\pi.
\end{equation}

If we use $f_\pi = 86$ MeV and $N = 3$, then we get $\bar{\mu}_8 = 1082 \pm 70$ MeV.

The vector meson masses have been computed in the quenched approximation for $N = 2, 3, 4, 6$ in [6, 7]. The couplings used in [6] and in [7] are roughly the same. The strongest and weakest coupling correspond to $b = 0.296$ and $b = 0.353$ respectively in the notation of this paper. There is a bulk transition on the lattice in the large $N$ limit that becomes a cross-over at finite $N$. The region between $b = 0.34$ and $b = 0.36$ is in the meta-stable region of this transition [4] and we need to be above $b = 0.34$ to be in the continuum phase of the large $N$ theory. Since the vector meson is heavy compared to the pion for small quark masses, finite lattice spacing effects are larger in the case of the vector meson. Our study at $b = 0.350$, not reported in this paper, does yield a value for $\bar{M}_8$ that is about 25% smaller than the one quoted here at $b = 0.360$ and consistent with the value obtained in [7].

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References