AST 105

Introduction to Astronomy: The Solar System

Announcement:
First Midterm this Thursday 09/24
Newton’s 3 Laws of Motion

1. An object moves at constant velocity if there is no net force acting on it.

2. When a force, $F$, acts on a body of mass $M$, it produces in it an acceleration, $A$, equal to the force divided by the mass. Or $A = \frac{F}{M}$.

3. For any force, there is always an equal and opposite reaction force.
Newton's Version of Kepler's Third Law

Orbital Time = Total Distance / Speed

\[ P = \frac{2\pi a}{V_{\text{circ}}} \]

\[ P^2 = \frac{(2\pi a)^2}{(V_{\text{circ}})^2} \]

\[ P^2 = \frac{4\pi^2 a^2}{(GM/a)} \]

\[ P^2 = \left\{ \frac{4\pi^2}{GM} \right\} a^3 \]

What is \( M \) again?

\( M = \text{Mass of the Orbit} \)
What if the orbitER and orbitEE are of similar masses?
Newton's Version of Kepler's Third Law

\[ p^2 = \frac{4\pi^2}{G(M_1 + M_2)} a^3 \]

- \( p \) = orbital period
- \( a \) = average orbital distance (between centers) = semi-major axis
- \((M_1 + M_2)\) = sum of object masses

General expression when \( M_1 \) and \( M_2 \) are comparable
What if the orbitER and orbitEE are of similar masses?

Pluto & Charon

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\( a \) = average orbital distance (between centers)

= semi-major axis

\((M_1 + M_2) = \text{sum of object masses}\)

- In most cases, \( M_1 + M_2 \approx M_1 \) (Mass of the bigger object)

General expression when \( M_1 \) and \( M_2 \) are comparable
In Newton’s version of Kepler’s 3rd Law \( p^2 = 4 \pi^2 a^3 / G(M_1 + M_2) \), why is \( M_1 + M_2 \) usually written as just \( M \)?

A. Because Newton’s version only works for one mass.
B. Newton’s version was designed for planets.
C. You only need the mass of the object you want the period of.
D. Because usually one of the masses is much smaller than the other.
E. The gravitational constant, \( G \), only contains kg, not kg\(^2\).
In Newton’s version of Kepler’s 3rd Law ($p^2=4\pi^2a^3/G(M_1+M_2)$), why is $M_1+M_2$ usually written as just $M$?

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C. You only need the mass of the object you want the period of.
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E. The gravitational constant, $G$, only contains kg, not kg$^2$. 
Kepler's 3rd Law

\[ P^2 = a^3 \]

ONLY works for orbiting the SUN

Newton's Version of Kepler's 3rd Law

\[ P^2 = \left\{ \frac{4\pi^2}{GM} \right\} a^3 \]

works for ANYTHING orbiting ANYTHING

\[ G = 6.67 \times 10^{-11} \text{ m}^3 / (\text{kg s}^2) \]
The mass of Jupiter can be calculated by

A. Knowing the Sun’s mass and measuring the average distance of Jupiter from the Sun

B. Measuring the orbital speed of one of Jupiter’s moons.

C. Measuring the orbital period and distance of Jupiter’s orbit around the Sun

D. Knowing the Sun’s mass and measuring how Jupiter’s speed changes during its elliptical orbit around the Sun

E. Measuring the orbital period and distance (from Jupiter) of one of Jupiter’s moons.
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C. Measuring the orbital period and distance of Jupiter’s orbit around the Sun
D. Knowing the Sun’s mass and measuring how Jupiter’s speed changes during its elliptical orbit around the Sun
E. Measuring the orbital period and distance (from Jupiter) of one of Jupiter’s moons.
Using Newton’s Version of Kepler’s 3rd Law (NVK3L)

\[ M = \frac{4\pi^2}{G} \frac{a^3}{P^2} \text{ kg} \]

Measure \( P \)

Timing: Count seconds to make 1 orbit

Measure \( a \)

Direct Observation, Parallax

Calculate \( M = \text{mass of orbit} \)

Three variables
If we know two, we can solve for the third!
How long would one orbit (for Earth) take if the Sun was 4 times as massive as it currently is?

A. 1/16th of a year (~3 weeks)
B. 1/4th of a year (3 months)
C. 1/2 of a year (6 months)
D. 1 year (12 months, 52 weeks, 365 days, $\pi \times 10^7$ seconds)
E. 2 years
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\[ p^2 = \frac{4\pi^2}{GM} a^3 \]

- \( M \) ↑ 4
- \( 1/M \) ↓ 4
- \( p^2 \) ↓ 4
- \( p \) ↓ √4 = 2
NVK3L can be used on anything that orbits!

Mass of Ida

\[
= \left\{ \frac{4\pi^2}{G} \right\} a^3 / P^2
\]

\(P \sim 1.5\) days
\(a \sim 100\) km

Ida (Asteroid)

Dactyl (asteroid moon)
NVK3L can be used on anything that orbits!

\[ M = \frac{4\pi^2}{G} \frac{a^3}{P^2} \]

\[ M = \frac{4\pi^2}{6.67 \times 10^{-11} \frac{m^3}{kg \cdot s^2}} \left( \frac{100km \cdot 1000m}{1km} \right)^3 \left( \frac{1.5day \cdot 86400s}{1day} \right)^2 \]

\[ M = 3.5 \times 10^{16} \text{ kg} \]
NVK3L can be used on anything that orbits!

Even if you can’t see the object in the center!
How to weigh a black hole…

• Watch stars orbit an invisible center
  - Mass of supermassive black hole = \( \frac{4\pi^2}{G} \alpha^3/P^2 \) ~\( \sim 4 \) millions of Suns!!
  \( P_{S0-2} \sim 16 \) years
  \( \alpha_{S0-2} \sim 1000 \) AU
Discovery of dark matter in the Milky Way

Sun’s velocity is about 220 km/sec

Sun’s velocity should be only 160 km/sec

Observed

Visible matter only

DARK MATTER by the dark matter halo.
Dark matter halo for galaxies

- Dark matter extends beyond visible part of the galaxy -- mass is $\sim 10x$ stars and gas!

- Probably not normal mass that we know of (protons, neutrons, electrons).

- Most likely subatomic particles, as yet unidentified!!! BIG MISTERY !!!!
Midterm I

• Covering Lectures Notes 2 through 9

• Strictly individual effort

Next: Midterm I review
Scales in the Universe: our Cosmic Address

Earth

Sun/Solar System

Milky Way Galaxy

Local Group

Local Supercluster
Lookback Time

• The image of a galaxy spreads across 100,000 years of time

• Try to think of what we SEE NOW as different from what may EXIST now
Daily Motions - Apparent

**Apparent motion** = Stars appear to move counterclockwise (when looking north)

Rise in the east - Set in the west
Daily Motions - Actual

Actual motion = Earth spins with axis pointed in fixed direction - towards Polaris

West to East: NY leads LA
Apparent Motion of the Sun

Equinox
**Do the planets reverse course?**

- Planets usually move slightly *eastward* from night to night (not in the course of one night!!) relative to the stars.
- But sometimes they go *westward* relative to the stars for a few weeks: apparent retrograde motion.
We see apparent retrograde motion when we pass by a planet in its orbit.
The REAL Reason for the Seasons
When the Sun is high in the sky, the amount of direct sunlight received is greater. This results in **SUMMER**

When the Sun is low in the sky, the amount of direct sunlight received is less. This results in **WINTER**
Although the Moon is always ½ lit by the Sun, we see different amounts of the lit portion from Earth depending on where the Moon is located in its orbit.
Phases of Moon

• **Moon is illuminated (always \( \frac{1}{2} \)) by Sun**

• **We see a changing combination of the bright and dark faces as Moon orbits the Earth**

See this demonstration: https://www.youtube.com/watch?v=MA2LON2QAA4
Lunar Eclipse

The Earth lies directly between the Sun and the Moon
Solar Eclipse

The Moon lies directly between the Sun and the Earth
The Science of Astronomy
Geocentric vs Heliocentric

Earth-Centered (Geocentric)

Sun-Centered (Heliocentric)
How did the Greeks explain planetary motion?

Plato

Aristotle

Greek geocentric model (c. 400 B.C.)
How did Copernicus challenge the Earth-centered idea?

Copernicus (early 1500s):

- Proposed Sun-centered (heliocentric) model
  - with circular orbits to determine layout of solar system

But . . .

- Model was no more accurate than Ptolemy's model in predicting planetary positions
Kepler first tried to match Tycho's observations with circular orbits.

Eventually he decided to try ellipses.

These orbits allowed him to match the data (and predict future observations) nearly perfectly.

"If I had believed that we could ignore those eight minutes of arc, I would have patched up my hypothesis accordingly. But, since this was not permissible to ignore, those eight minutes pointed the road to a complete reformation in astronomy."
Kepler's First Law: The orbit of each planet around the Sun is an ellipse with the Sun at one focus.
**Kepler's Second Law:** As a planet moves around its orbit, it sweeps out equal areas in equal times.

The areas swept out during all equal time periods are equal.
Kepler's Third Law: Planetary orbits follow the mathematical relationship:

\[ p^2 = a^3 \]

- \( p \) = orbital period in years
- \( a \) = avg. distance from Sun in AU
- semi-major axis of elliptical orbit

*Planet's mass doesn't matter!*
Newton’s 3 Laws of Motion

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2. When a force, $F$, acts on a body of mass, $M$, it produces in it an acceleration, $A$, equal to the force divided by the mass. Or $A = F/M$.

3. For any force, there is always an equal and opposite reaction force.
Universal Law of Gravity

\[ F_g = G \frac{M_1 M_2}{d^2} \]

\[ G = 6.67 \times 10^{-11} \text{ m}^3/(\text{kg s}^2) \]

"BIG G"
Acceleration Due to Earth's Gravity

\[ F = G \frac{M_1 M_2}{d^2} \]
\[ A = F/M \]

\[ A_c = \frac{G M_{\text{Earth}}}{R_{\text{Earth}}^2} \frac{M_{\text{cabbage}}}{M_{\text{cabbage}}} \]

\[ M_1 = M_{\text{Earth}} \]
\[ d = \text{radius of Earth} = R_E \]
**How is mass different from weight?**

- **Mass** – the amount of matter in an object
- **Weight** – the force that acts upon an object (based on acceleration and mass)

Your weight can change a lot depending on where you are but your mass doesn't.
Orbital Speed

• Planets (orbiters) are constantly trying to get away
  - Gravity (from the orbitee) is constantly pulling them back

• We can use Newton’s law of gravity to calculate how fast they move

\[ V_{\text{circular}} = \sqrt{\frac{GM_{\text{orbitee}}}{r}} \]
**Escape Velocity**

The velocity needed to escape the gravity of the orbit EE

**Earth**

- $V_{\text{circ}} = 8 \text{ km/s}$
- $V_{\text{esc}} = 11 \text{ km/s}$

**Equations**

- Escape velocity: $V_{\text{escape}} = \sqrt{\frac{2GM_{\text{orbit}}}{r}}$
- Orbital velocity: $V_{\text{circular}} = \sqrt{\frac{GM_{\text{orbit}}}{r}}$