RELATIVITY OF SPACE AND TIME IN POPULAR SCIENCE

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x₀: 7:30p EST
x¹: Flatiron Institute, Center for Computational Astrophysics
RELATIVITY OF SPACE AND TIME IN POPULAR SCIENCE: TALK OUTLINE

1. Popular Science Jeopardy

2. Unpopular Science (lots of tensors):
   a) Electromagnetism
   b) Special Relativity
   c) General Relativity

3. Fate of the Universe
   • References:
     • *Interstellar* (2014) - Directed by Christopher Nolan, *Interstellar* portrays a bleak future of a climate-change-ravaged Earth and a daring mission led by Cooper’s team to chart out escape plans in a planetary system around the supermassive black hole Gargantua.
     • *The Time Machine* (1895) – Authored by H. G. Wells, this classic tome portrays a gripping class conflict nearly a million years into the future of Victorian England, where our ancestors split into two unrecognizable classes—with both upper and lower appearing completely devoid of humanity
<table>
<thead>
<tr>
<th>Are We Alone?</th>
<th>Time Travel</th>
<th>Astronomy</th>
<th>Astrology</th>
<th>Cosmology or Cosmetology?</th>
<th>To Infinity and Beyond!</th>
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JEOPARDY – TIME TRAVEL
JEOPARDY – ARE WE ALONE
JEOPARDY – ASTRONOMY
JEOPARDY – FINAL JEOPARDY
MATH METHODS – VECTOR CALCULUS

- Vectors in 3D may be multiplied using
  - Dot product: \( \vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z = ab \cos \theta \)
  - Cross product: \( \vec{a} \times \vec{b} = (a_y b_z - b_y a_z) \hat{x} + (a_z b_x - b_z a_x) \hat{y} + (a_x b_y - b_x a_y) \hat{z} \)

- A vector field \( \mathbf{F}(x,y,z) \) is specified by assigning a vector to each point in space. At each point we may calculate
  - Divergence (whether the vector field looks like a source or a sink)
    \[ \hat{\nabla} \cdot \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \]
  - Curl (whether a paddlewheel would rotate in the vector field)
    \[ \hat{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial F_x}{\partial x} & \frac{\partial F_x}{\partial y} & \frac{\partial F_x}{\partial z} \\ \frac{\partial F_y}{\partial x} & \frac{\partial F_y}{\partial y} & \frac{\partial F_y}{\partial z} \\ \frac{\partial F_z}{\partial x} & \frac{\partial F_z}{\partial y} & \frac{\partial F_z}{\partial z} \end{vmatrix} \]
ELECTROMAGNETISM – MAXWELL’S EQUATIONS

- Maxwell’s equations describe the dynamics of electric and magnetic fields

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} )</td>
<td>Gauss’s Law</td>
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<tr>
<td>( \nabla \cdot \vec{B} = 0 )</td>
<td>Gauss’s Law for Magnetism</td>
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<td>( \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} )</td>
<td>Faraday’s Law</td>
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<td>( \nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} )</td>
<td>Ampère Law</td>
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Lorentz Force Law: \( \vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \)

Now that we know these basic relationships between \( \vec{E} \) and \( \vec{B} \) ... Let there be light!

Poynting flux (power per unit area): \( \vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \)

Let’s also see Maxwell’s Eqs. in integral form
## ELECTROMAGNETISM – MAXWELL’S EQUATIONS

- Rewriting Maxwell’s equations in integral form

<table>
<thead>
<tr>
<th>Differential Form</th>
<th>Integral Form</th>
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<tbody>
<tr>
<td>( \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} )</td>
<td>( \oiint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0} )</td>
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<tr>
<td>( \nabla \cdot \vec{B} = 0 )</td>
<td>( \oiint \vec{B} \cdot d\vec{A} = 0 )</td>
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<td>( \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} )</td>
<td>( \oint \vec{E} \cdot d\vec{l} = -\frac{\partial \Phi_B}{\partial t} )</td>
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<tr>
<td>( \nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} )</td>
<td>( \oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{\partial \Phi_E}{\partial t} )</td>
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</table>
In 1905 Albert Einstein (1879-1955) wrote observations from a series of thought experiments and observations in *On the Electrodynamics of Moving Bodies*:

- Relative, not absolute motion, establishes current in magnet-conductor system
- Simultaneity depends on an observer’s state of motion, or reference frame
- Clocks synchronize if the difference in times (measured locally) from when light is emitted at A to when light arrives at B equals the corresponding difference for the reverse journey.
In the frame of the conductor, the moving magnet creates a changing magnetic flux that results in an electric field in the conductor, and subsequently a current.

In the frame of the magnet...

\[
\vec{E} = -\frac{\partial \Phi_B}{\partial t}
\]

\[
\int \vec{E} \cdot d\vec{l} = -\frac{\partial \Phi_B}{\partial t}
\]
In the frame of the conductor, the moving magnet creates a changing magnetic flux that results in an electric field in the conductor, and subsequently a current.

\[ \int \vec{E} \cdot d\vec{l} = -\frac{\partial \Phi_B}{\partial t} \]

In the frame of the magnet, the moving conductor’s free charges are deflected by the magnetic field.

\[ \vec{F}_B = q\vec{v} \times \vec{B} \]
POSTULATES OF SPECIAL RELATIVITY

• Postulate 1: “The laws by which the states of physical systems undergo change are not affected, whether these changes of state be referred to the one or the other of two systems of co-ordinates in uniform translatory motion” (Einstein, 1905, p.4).

The laws of physics are the same in any two inertial frames

• Postulate 2: “Any ray of light moves in the ‘stationary’ system of co-ordinates with the determined velocity c, whether the ray be emitted by a stationary or by a moving body” (Einstein, 1905, p.4).

The speed of light in a vacuum is the constant c regardless of the motion of the emitting frame

\[ c = 299,792,458 \text{ m/s} \]
REFERENCE FRAMES

- Frame S is a set of space and time coordinates with Origin O at (0,0,0,0).
- At other points $r$ in S, clocks at rest in S are synchronized with the origin by accounting for light travel time $r/c$ (simultaneous events at O and $r$ register $r/c$ later on clocks at $r$).
TRANSFORMATIONS – VECTOR ROTATIONS

- At $t=0$, imagine rotating 3-vector $\mathbf{r}$ into $\mathbf{r}'$: $\mathbf{r}' = A\mathbf{r}$

- This can be written

$$ r_i' = \sum_{j=1}^{3} A_{ij} r_j $$

$$ A(\theta) = \begin{bmatrix} \cos \theta + n_x^2(1 - \cos \theta) & n_x n_y(1 - \cos \theta) - n_z \sin \theta & n_x n_z(1 - \cos \theta) + n_y \sin \theta \\ n_y n_x(1 - \cos \theta) + n_z \sin \theta & \cos \theta + n_y^2(1 - \cos \theta) & n_y n_z(1 - \cos \theta) - n_x \sin \theta \\ n_z n_x(1 - \cos \theta) - n_y \sin \theta & n_z n_y(1 - \cos \theta) + n_x \sin \theta & \cos \theta + n_z^2(1 - \cos \theta) \end{bmatrix} $$

$$ A^{-1} = A(-\theta) = A^T $$

Length invariant under rotation:

$$ |A\mathbf{r'}|^2 = (A\mathbf{r'})^T (A\mathbf{r'}) = \mathbf{r'}^T A^T A \mathbf{r'} = \mathbf{r'}^T A^{-1} A \mathbf{r'} = \mathbf{r'}^T \mathbf{r'} = |\mathbf{r'}|^2 $$
LORENTZ TRANSFORMATIONS AND INERTIAL FRAMES

- Inertial frames move relative to each other at constant speed, without acceleration.
- In relativity, space and time coordinates of an event observed in different inertial frames are related by Lorentz transformations: $t \rightarrow t'(t,x,y,z)$, $x \rightarrow x'(t,x,y,z)$, $y \rightarrow y'(t,x,y,z)$, $z \rightarrow z'(t,x,y,z)$.
- Frame $S'$ is said to be “boosted” with velocity $(v,0,0)$ with respect to $S$, and has Origin $O'$ at $(t',vt',0,0)$.

Vector $r'$ in $S'$ is related to $r$ in $S$ via Lorentz transformation.

$x, x'$
LORENTZ TRANSFORMATIONS

• Space and time components are mixed by Lorentz boost $\Lambda_{\mu}^{\nu}$ in the direction $\hat{n} = (n_x, n_y, n_z)$ to another inertial (non-accelerating) frame, where

\[
\Lambda_{\mu}^{\nu} = \begin{pmatrix}
\gamma & -\gamma \beta n_x & -\gamma \beta n_y & -\gamma \beta n_z \\
-\gamma \beta n_x & 1 + (\gamma - 1)n_x^2 & (\gamma - 1)n_x n_y & (\gamma - 1)n_x n_z \\
-\gamma \beta n_y & (\gamma - 1)n_y n_x & 1 + (\gamma - 1)n_y^2 & (\gamma - 1)n_y n_z \\
-\gamma \beta n_z & (\gamma - 1)n_z n_x & (\gamma - 1)n_z n_y & 1 + (\gamma - 1)n_z^2 
\end{pmatrix}
\]

• The Lorentz transformation of 4-vector $x^\mu$ into $(x')^\mu$ takes the form

\[
(x^\mu)' = \Lambda_{\mu}^{\nu} x^\nu
\]

In Einstein summation notation, an index repeated upstairs/downstairs (or vice versa) indicates a sum over that index
FOUR VECTORS

\[ \vec{J} = (c \rho, \vec{J}) \]

\[ \vec{P} = \left( \frac{E}{c}, \vec{p} \right) \]

\[ \vec{K} = \left( \frac{\omega}{c}, \vec{k} \right) \]

\[ u^\mu = \frac{d x^\mu}{d \tau} = \gamma \frac{d x^\mu}{d t} = (\gamma c, \gamma \vec{v}) \]

\[ x^\mu = (ct, \vec{x}) \]
LORENTZ TRANSFORMATIONS - EXAMPLE

• For a boost of velocity \( \mathbf{v} = (v,0,0) \) of Frame S’ relative to Frame S, what is the Lorentz transformation tensor, and how does the 4-displacement \((x^0, x^1, x^2, x^3)\) transform?

\[
\Lambda_{\mu}^{\nu} = \begin{pmatrix}
\gamma & -\gamma \beta & 0 & 0 \\
-\gamma \beta & \gamma & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

\[
(x^0)' = \gamma x^0 - \gamma \beta x^1 \\
(x^1)' = -\gamma \beta x^0 + \gamma x^1 \\
(x^2)' = x^2 \\
(x^3)' = x^3
\]
• What are fundamental differences between space and time dimensions? Consider p. 6:

‘But,’ said the Medical Man, staring hard at a coal in the fire, ‘if Time is really only a fourth dimension of Space, why is it, and why has it always been, regarded as something different? And why cannot we move in Time as we move about in the other dimensions of Space?’

• Unlike other spacetime dimensions, time has a fixed direction (arrow of time) in which events progress, as seen in thermodynamics.
WORLDLINES, WORLDSHEETS, WORLDVOLUMES

- The worldline of a particle is the path generated by its trajectory in space and time (times the speed of light $x_0 = ct$)
- The worldsheet is the generalization of a worldline to trajectories of 1-D objects, developed by Leonard Susskind to describe open and closed strings. Worldvolumes are higher dimensional generalizations.

The worldlines of one particle at rest in S, and another accelerating in the positive, then negative x-direction
TIME DILATION

• Compare a light clock that undergoes a tick (round trip of a photon) in Frames S and S’
• Special relativity postulates the speed of light is the same in both stationary and moving clock
• During a tick $\Delta t'$ of the moving clock, the path length of light’s worldline is longer in $S'$ than in $S$

\[
\Delta t = \frac{2h}{c}
\]

\[
\Delta t' = \frac{2\sqrt{h^2 + (\sqrt{\Delta t^2 / 2})^2}}{c}
\]

\[
\Rightarrow \Delta t' = \gamma \Delta t, \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}
\]
PROPER TIME ON A WORLDLINE

• The time measured between events in a frame in which clocks are stationary is the proper time $\tau$, and time dilation reads

$$dt' = \gamma d\tau$$

• On Worldline AB connecting Event A to Event B, the proper time is

$$\tau_{AB} = \int_A^B \sqrt{1 - v^2(t')/c^2} dt'$$
THE TIME MACHINE - PROPER TIME AND 4-VELOCITY

• Classically, the rate at which an object travels through space is $dx/dt$, but there is no natural way to express a rate of passage through time.

• Consider p. 15

delightfully. We cannot see it, nor can we appreciate this machine, any more than we can the spoke of a wheel spinning, or a bullet flying through the air. If it is travelling through time fifty times or a hundred times faster than we are, if it gets through a minute while we get through a second, the impression it creates will of course be only one-fiftieth or one-hundredth of what it would make if it were not travelling in time. That's plain

• How can we use relativity to describe the rate of passage through space AND time?

• In relativity, we can write the rate of travel through space AND through time as $dx^i/d\tau = \gamma v$ and $dx^0/d\tau = \gamma c$

\[
dx^\mu = (dx^0, d\vec{x}) = (cd\tau, d\vec{x}) = (\gamma c d\tau, d\vec{x})
\]

\[
\gamma^\mu = \frac{dx^\mu}{d\tau}
\]
LENGTH CONTRACTION

• Now consider Events A, B and C for light bouncing parallel to the motion of a light clock in Frames $S'$

\[
t'_{AB} = ?
\]

\[
t'_{BC} = ?
\]
LENGTH CONTRACTION

• Now consider Events A, B and C for light bouncing parallel to the motion of a light clock in Frames S’.
• Imagine entering a reference frame comoving with a spaceship
POP QUIZ - RELATIVITY OF SIMULTANEITY

- If two equidistant light sources turn on at the same time in the spaceship comoving frame, do you first see light from
  a) Source A
  b) Source B
  c) Both Sources A and B
POP QUIZ - RELATIVITY OF SIMULTANEITY

Now enter a reference frame in which the spaceship is moving at velocity $v$. Does the person in the rocket first see

a) Source A
b) Source B

c) Both Sources A and B
We have seen in special relativity that distances, time intervals and even simultaneity depend on the inertial reference frame in which they are measured.

Is anything invariant under Lorentz transformation?

The spacetime interval

\[(\Delta S')^2 = -(\Delta x^0')^2 + (\Delta x^1')^2 + (\Delta x^2')^2 + (\Delta x^3')^2 = (\Delta S)^2\]

is Lorentz invariant
• Interstellar Min (39:10-40:50) - Cooper explains Murph will think his watch slow when he is traveling close to the speed of light (or, as we’ll seem later, when he is near a black hole) in his mission. To what relativistic phenomenon does this pertain?
• Interstellar Min (39:10-40:50) - Cooper explains Murph will think his watch slow when he is traveling close to the speed of light or is near a black hole in his mission due to time dilation.
BOHR CORRESPONDENCE PRINCIPLE

• A revised theory must agree with the previously established theory in the classical limit
RELATIVISTIC CORRECTIONS TO CLASSICAL PHYSICS – VELOCITY ADDITION

• In relativity, no object or information can travel faster than light, a fact reflected in the relativistic law of composition of velocities

\[ v_{AC} = \frac{v_{AB} + v_{BC}}{1 + \frac{v_{AB}v_{BC}}{c^2}} \]

• \( v_{AB} \) is the (1D) velocity of A with respect to B
• \( v_{BC} \) is the (1D) velocity of B with respect to C
POP QUIZ - RELATIVISTIC CORRECTIONS TO CLASSICAL PHYSICS – VELOCITY ADDITION

\[ v_{AC} = \frac{v_{AB} + v_{BC}}{1 + \frac{v_{AB}v_{BC}}{c^2}} \]

\[ v_{AC} = 0.9c \]

\[ v_{BC} = 0.5c \]

- Compare \( v_{AC} \) to \( c \)
- What should \( v_{AC} \) reduce to in the non-relativistic limit (\( v_{AB}, v_{BC} \ll c \)) in terms of \( v_{AB} \) and \( v_{BC} \)?
POP QUIZ - RELATIVISTIC CORRECTIONS TO CLASSICAL PHYSICS – VELOCITY ADDITION

- Compare $v_{AC}$ to $c$.
- What should $v_{AC}$ reduce to in the non-relativistic limit ($v_{AB}$, $v_{BC} << c$) in terms of $v_{AB}$ and $v_{BC}$?

$$v_{AC} = \frac{v_{AB} + v_{BC}}{1 + \frac{v_{AB}v_{BC}}{c^2}}$$

$$v_{AC} = \frac{0.9 + 0.5}{1 - 0.9 \cdot 0.5} \cdot c = 0.9655c < c$$

$$v_{AC} \to v_{AB} + v_{BC}$$
• Length contraction
  • Exercise: A circle of radius $r=1$ in its rest frame is boosted with $\gamma=2$. What is its area now?
• Relativity of simultaneity
  • Whether two events are simultaneous depends on the state of motion of the observer
GENERAL THEORY OF RELATIVITY

- Einstein sought to extend his 1905 special theory of relativity to include non-inertial (accelerating) frames.

- In strong gravitational fields, such as near black holes or neutron stars, classical + special relativistic predictions for lengths, times, frequencies, energy, etc. fail.

- In a 1915 lecture at the Prussian Academy of Sciences in Berlin, Einstein unified gravity with special relativity in the general theory of relativity.
EQUIVALENCE PRINCIPLE

• Consider two labs: GraviLab and AccLab
• Is there any experiment in either lab that can distinguish the black hole from the piston?
EQUIVALENCE PRINCIPLE - TIDAL FORCES

• Consider two labs: GraviLab and AccLab
• Tidal forces would affect only the person in GraviLab:

\[ T \approx \frac{MH}{r^3} \]
EQUIVALENCE PRINCIPLE

• Consider two labs: GraviLab and AccLab
• For sufficiently small labs, there is no experiment to distinguish them
EQUIVALENCE PRINCIPLE

- Equivalence principle:

Local properties of curved spacetime are indistinguishable from flat spacetime

- Mathematically, near any point $P$ of globally curved spacetime there exists a local coordinate transformation of the metric tensor $g_{\mu\nu} \rightarrow g_{\alpha\beta}$ transforming it to the flat space metric tensor

$$g_{\alpha\beta}(P) = \eta_{\alpha\beta}$$
INTERSTELLAR (1:08:25-1:10:40) - TIDAL FORCES

• Interstellar Min 1:09:18 - Miller (Water) Planet 130% Earth gravity
• Interstellar Min 1:10:30 - Miller (Water) Planet mountainous waves
In 1783 John Mitchell theorized that if a star’s radius is $>500R_{\text{Sun}}$, light could not escape it.

In 1916, Karl Schwarzschild solved the Einstein field equations for the geometry outside a spherically symmetric, non-rotating mass.

In 1963, Roy Kerr solved the Einstein field equations for the geometry outside a spherically symmetric, rotating mass.

In the 1970's, black hole temperature was theorized by Hawking, among others.
STRONG GRAVITY AND BLACK HOLES

• We have seen that Einstein’s special relativity postulates can warp space and time
• Can gravity warp spacetime as well?
  • Yes! Dense objects change the spacetime distance formula from flat space
  • STRONG gravity as one approaches singularity of infinite spacetime curvature
Objects launched from massive bodies will fall back unless their kinetic energy is at least the magnitude of the potential energy binding them to the object:

\[ \frac{m v_{\text{esc}}^2}{2} = \frac{G M m}{R} \]

\[ v_{\text{esc}} = \sqrt{\frac{2 G M}{R}} \]

- Note: The object’s mass \( m \) does not enter the escape speed.

- **Exercise:** Apply the escape speed condition to a massless photon traveling at the speed of light \( c \) to find the Schwarzschild radius \( r_s \), i.e., the radius in which \( M \) would have to be contained so that light cannot escape.
Black Holes - Theory

- Objects launched from massive bodies will fall back unless their kinetic energy is at least the magnitude of the potential energy binding them to the object:

\[
\frac{mv_{\text{esc}}^2}{2} = \frac{G M m}{R}
\]

\[
\Rightarrow v_{\text{esc}} = \sqrt{\frac{2 G M}{R}}
\]

- Note: The object's mass \( m \) does not enter the escape speed

- Exercise: Apply the escape speed condition to a massless photon traveling at the speed of light \( c \) to find the Schwarzschild radius \( r_s \), i.e., the radius in which \( M \) would have to be contained so that light cannot escape.

\[
c = \sqrt{\frac{2 G M}{R}}
\]

\[
\Rightarrow r_s = R = \frac{2 G M}{c^2}
\]
INTERSTELLAR (1:08:25-1:10:40 & 1:46:15-1:47:28) - TIDAL FORCES

- Interstellar Min 1:11:00 - Miller (Water) Planet 130% Earth gravity
- Interstellar Min 1:11:45 - Miller (Water) Planet mountainous waves
- Interstellar 1:46:40 Romiley explains Gargantua is a "gentle" enough singularity that a probe crossing the horizon could survive
GRAVITATIONAL TIME DILATION AND REDSHIFT

- Gravitational time dilation
  - Gravitational fields increase the oscillation period of an escaping photon and slow clocks according to

\[
\frac{t_0}{t_d} = \sqrt{1 - \frac{v_{\text{esc}}^2}{c^2}} = \frac{1}{\sqrt{1 - \frac{2GM}{rc^2}}} = \frac{1}{\sqrt{1 - \frac{r_s}{r}}} = \frac{1}{\sqrt{1 - \frac{\hbar}{E_S}}}
\]

- **Exercise:** A visible photon leaving the vicinity of a black hole appears to
  - (a.) get redder
  - (b.) get bluer
  - (c.) stay the same

\[
E_\gamma = hf = \frac{h}{T} = \frac{hc}{\lambda}
\]

As the photon loses energy, \( \lambda \) increases, resulting in gravitational redshift.
GENERAL RELATIVITY SUMMARY

• General relativity
  • Gravity warps spacetime:
    \[ ds^2 = g_{\mu\nu}dx^\mu dx^\nu \]
  • Equivalence principle: Local properties of curved spacetime are indistinguishable from flat spacetime (modulo tidal forces)
    \[ g_{\alpha\beta}(P) = \eta_{\alpha\beta} \]

• Gravitational time dilation
  • Gravitational fields increase the oscillation period of an escaping photon and slow clocks according to
    \[ t_0 = t_d \sqrt{1 - \frac{2GM}{rc^2}} = t_d \sqrt{1 - \frac{r_s}{r}} \]
BLACK HOLES IN ASTRONOMY

• Remnants of some stars $\gtrsim 4M_{\text{Sun}}$ produce black holes when the star runs out of nuclear fuel providing outward pressure against gravitational collapse

• If a black holes is formed from a star that was in a binary system, it accretes the companion star, producing x-ray radiation

• Supermassive ($10^6$-$10^{10}M_{\text{Sun}}$) black holes reside in Active Galactic Nuclei

• Relativistic jets of radiating cosmic rays can be ejected from the poles of black holes in:
  • Active galactic nuclei
  • BH/X-Ray binaries
  • Gamma ray bursts
APPROACHING THE HORIZON

Sgr A*

Zooming in on Milky Way’s Sgr A* Region
Courtesy of European Southern Observatory (ESO)
BLACK HOLES - OBSERVATIONS

- The Event Horizon Telescope is a collection of radio antennae forming a network of intercontinental baselines to form mm-images.

- **Exercise:** Can you explain why baselines of radio telescopes are so long in view of the angular resolution limit below?
  - The ability to distinguish two sources at smaller angular separation increases with aperture diameter.

\[
\Delta \theta_{\text{min}} = 1.22 \frac{\lambda}{D_{\text{aperture}}}
\]
INTERSTELLAR-EINSTEIN RINGS

- Interstellar 2:13:24 – Shuttle approaches Gargantua with visible Einstein rings
HUBBLE’S LAW

• Hubble observed light from galaxies was far more likely to be redshifted than blueshifted, as they were receding into an expanding universe with speed as a linear function of distance:

\[ v = H_0 D, \quad H(t) = \frac{\dot{a}}{a(t)} \]

Scale factor \( a(t) \) compares lengths comoving with fabric of space to initial lengths in a dynamic universe.

• The current value of the Hubble constant (which is constant over space, not time) is

\[ H_0 = 70 \, \frac{\text{km/s}}{\text{Mpc}} \]

indicating that for every megaparsec one looks into the sky, objects recede an additional 70km/s.
FATE OF THE UNIVERSE

• Solve the 1st Friedmann equation

\[ H^2 + \frac{kc^2}{a^2} - \frac{c^2}{3} \Lambda = \frac{8\pi G}{3} \rho, \quad H = \frac{\dot{a}}{a} \]

for the scale factor \( a(t) \) in an expanding \( \Lambda \)-dominated Universe.

\[ H = \pm c \sqrt{\frac{\Lambda}{3}} \]

\[ \Rightarrow a = \frac{da}{dt} = Ha = \pm c \sqrt{\frac{\Lambda}{3}} a \]

\[ \Rightarrow a(t) \sim e^{Ht} = e^{\pm c \sqrt{\frac{\Lambda}{3}} t} \]
FATE OF THE UNIVERSE

- The fate of the universe depends on the ratio $\Omega = \frac{\rho}{\rho_{\text{crit}}}$ of its density relative to the critical density.
  - If the Universe has $\Omega < 1$, then it will eventually contract under its own gravity into a fiery collapse.
  - If the Universe has $\Omega = 1$, then it will continue expanding indefinitely, but at an ever-slowing rate.
  - If the Universe has $\Omega > 1$, then its expansion will accelerate into a cold, isolated future.

\[
H^2 + \frac{k c^2}{a^2} - \frac{c^2}{3} \Lambda = \frac{8\pi G}{3} \rho
\]

1st Friedmann Equation

\[
H = \frac{\dot{a}}{a}, \quad H_0 = 70 \frac{\text{km/s}}{\text{Mpc}}
\]

\[
\rho_M = \Omega_M \rho_{\text{crit}}, \quad \rho_r = \Omega_r \rho_{\text{crit}}, \quad \rho_\Lambda = \Omega_\Lambda \rho_{\text{crit}},
\]

\[
\Omega = \Omega_M + \Omega_r + \Omega_\Lambda, \quad \Omega_{\text{crit}} = 1
\]

\[
\rho_{\text{crit},0} = 9.47 \times 10^{-27} \frac{\text{kg}}{\text{m}^3}
\]
A. Reiss (JHU), B. Schmidt (Australia National U.) and S. Perlmutter (Berkeley) won the 2011 Nobel Prize in physics for observing the accelerating expansion of the Universe. Popular science summary given in: https://www.nobelprize.org/nobel_prizes/physics/laureates/2011/popular-physicsprize2011.pdf

\[
H^2 + \frac{kc^2}{a^2} - \frac{c^2}{3}\Lambda = \frac{8\pi G}{3}\rho \\
\Omega = \Omega_M + \Omega_r + \Omega_\Lambda, \quad \Omega_{\text{crit}} = 1 \\
\rho_{\text{crit}} = 9.47 \times 10^{-27} \text{ kg/m}^3
\]

Brightness (mag) vs. distance (z) for Type Ia supernovae from observations by Brian Schmidt’s High-z Supernova Search Team (Riess et al. 1998) and Saul Perlmutter’s Supernova Cosmology Project (Perlmutter et al. 1999). Theoretical curves overlay the observations for cosmological models \((\Omega_M, \Omega_\Lambda) = (1.0, 0.0), (0.3, 0.0), (0.3, 0.7)\). The best fit is for the \(\Lambda\)-dominated Universe.
THANK YOU!