Outline

• Course administration
  – final presentations: select topics today
  – problem set 3

• Midterm exam
  – next Wednesday, April 1, 10:40–11:35am
  – review

• Review of previous lecture
  – exoplanet search techniques; direct imaging
Example Presentation Topics

• Nearby young brown dwarfs (Josh Schlieder)
• Y dwarfs: empirical and theoretical expectations
• The 2MASS J0535–0546 brown dwarf eclipsing binary: why is the less massive component hotter?
• The substellar initial mass function: comparison among star forming regions and implications for the formation of brown dwarfs
• Non-equilibrium chemistry in substellar atmospheres
• Magnetic activity and rotation in low-mas stars and brown dwarfs
• Nemesis: Sun's hypothetical binary companion
• The origin of hot Jupiters
• The Kozai mechanism and the eccentricities of exoplanet orbits
• Searching for exo-earths through gravitational microlensing
• Planet formation through core accretion
• Planet formation through gravitational disk instability
• Planet migration and the architecture of planetary systems
• Extremely high contrast imaging of exoplanets: present and future
• Dynamical signatures of unseen planets in optically thin circumstellar debris disks
Outline

• Course administration
  – final presentations: select topics today
  – problem set 3

• Midterm exam
  – next Wednesday, April 1, 10:40–11:35am
  – review

• Review of previous lecture
  – exoplanet search techniques; direct imaging
Review Topics

- binary stars: dynamical masses
- stellar and substellar interiors; evolution
- nuclear fusion in the cores low-mass objects
- spectral energy distributions, dominant atmospheric absorbers, spectral classification
- stellar and substellar photospheres
  - effective temperature, surface gravity, metallicity, evolution
- dust and clouds in substellar atmospheres
- hot Jupiters:
  - temperature structure and dynamics of atmospheres
  - radii; Rossiter-McLaughlin effect
Binary Star Dynamical Masses

- **Resolved visual binaries**: see stars separately, measure orbital axes and speeds directly.

- **Astrometric binaries**: only brighter member seen, with periodic wobble in the track of its proper motion.

- **Spectroscopic binaries**: unresolved (relatively close) binaries told apart by periodically oscillating Doppler shifts in spectral lines. Periods = days to years.
  - **Eclipsing binaries**: orbits seen nearly edge on, so that the stars actually eclipse one another. (Most useful.)
Dynamical Mass Determination

– If orbital major axes (relative to center of mass) or radial velocity amplitudes are known, so is the ratio of masses:

\[
\frac{m_1}{m_2} = \frac{a_2}{a_1} = \frac{v_{2r}}{v_{1r}}
\]

– If the period, \( P \), and the sum of major axis lengths, \( a = a_1 + a_2 \), are known, Kepler’s third law can give masses separately:

\[
P = \left( \frac{4\pi^2}{G(m_1 + m_2)} a^3 \right)^{1/2}
\]
Internal Equilibrium Equations

- hydrostatic equilibrium
  \[ \frac{dP}{dr} = - \frac{GM_r \rho}{r^2} \]

- mass continuity
  \[ \frac{dM_r}{dr} = 4\pi r^2 \rho \]

- conservation of energy
  \[ \varepsilon_v \] — energy emitted in neutrinos
  \[ \frac{dL_r}{dr} = 4\pi r^2 \rho (\varepsilon - \varepsilon_v) \]

- temperature continuity
  — depends on mode of energy transport
Modes of Energy Transport and the Temperature Continuity Equation

- **radiation**
  - photons absorbed by cooler outer layers
  - efficient in:
    - >1 $M_{\text{Sun}}$ star envelopes
    - cores of 0.3–1 $M_{\text{Sun}}$ stars
    - all stellar photospheres

- **convection**
  - adiabatic exponent $\gamma = C_P/C_V$
  - important when radiation inefficient:
    - interiors of brown dwarfs and <0.3 $M_{\text{Sun}}$ stars
    - cores of >1 $M_{\text{Sun}}$ stars
    - envelopes of ~1 $M_{\text{Sun}}$ stars

\[
\frac{dT}{dr} = -\frac{3k\rho L_r}{64\pi r^2 \sigma T^3}
\]

\[
\frac{dT}{dr} = \left(1 - \frac{1}{\gamma}\right) T \frac{dP}{dr}
\]
Hydrogen phase diagram

Evolution is towards:
- lower entropy $S$
- higher degeneracy $\eta$

$T \propto \rho^{0.67}$
$T \propto \rho^{0.4}$

(Burrows & Liebert 1993)
A Brown Dwarf’s and Jupiter’s Interiors

\[ P \approx 5 \text{ bar}, T \approx 1000 \text{ K}, \rho \approx 10^{-4} \text{ g/cc} \]

\[ P \approx 10^{11} \text{ bar}, T \approx 10^6 \text{ K}, \rho = 500 \text{ g/cc} \]

0.05 \( M_{\text{Sun}} \)

brown dwarf

\( (4 \times 10^9 \text{ bar}) \)

\( (4 \times 10^9 \text{ bar}) \)

\( (1 \text{ bar}) \)

Molecular \( \text{H}_2 \) (\( Y \approx 0.23 \))

Inhomogeneous?

Metallic \( \text{H} \) (\( Y \approx 0.27 \))

\( 15000-21000 \text{ K} \)

\( 4000 \text{ GPa} \)

Ices + Rocks core?

\( 165-170 \text{ K} \)

\( 100 \text{ kPa} \)

\( (1 \text{ bar}) \)

\( \text{Guillot 2006} \)
Radius vs. Mass: Comparison with Known Planets

- for polytropes
  \[ R \propto M^{3-n} \]
  \[ R \propto M^{1-n} \]

- \( n = 1.5 \) for brown dwarfs
- \( n = 0.5-1.0 \) for 0.1–1 \( M_{\text{Jup}} \) planets
- \( (n = 0: \text{uniform density}) \)

- icy/rocky cores in Neptune, Uranus?

\( \text{(Guillot 2006)} \)
p-p Chain Reaction Rate in Low-Mass Stars Is Decided by the First Step (p+p)

take into account Coulomb coupling
Minimum Main Sequence Mass

\[ M_{\text{MMSM}} = 0.0865 \, M_{\text{Sun}} \left( \frac{10^{-2} \, \text{g cm}^{-2}}{\kappa_R} \right) \frac{I(\eta)}{I(\eta_{\text{min}})} \]

\[ I(\eta) = \frac{(\eta + \alpha)^{1.509}}{\eta^{1.325}} \]

- \( M_{\text{MMSM}} \) depends on:
  - opacity \( \kappa_R \) (i.e., metallicity \( Z \))
  - He content \( Y \) (through \( \alpha(Y) \))
- \( M_{\text{MMSM}} = 0.075 \, M_{\text{Sun}} \) at solar \( Y \) (25\%), \( Z \) (1.6\%)
  - lower for higher \( Y, Z \)
Li and D: Depleted within Few 100Myr

(Burrows et al. 2001)
The Optical to IR SEDs of UCDs (Cushing et al. 2006; Marley & Leggett 2008)
SEDs: Near-IR Wavelengths

- reddish L dwarf colors due to dust in the visible atmosphere
- neutral T dwarf colors due to dust-free atmosphere, molecular opacity
- 2MASS $J - K_s$ colors:
  - M5: $\sim$0.9 mag
  - L5: $\sim$1.7 mag
  - T6: $\sim$0 mag
Near-IR CMD of Stars and Brown Dwarfs

(Kirkpatrick 2005)
Luminosity (i.e., Surface Gravity)

Effects at A0

(figure: D. Gray)
From Lecture 5: Line Profiles

- Natural line width (Lorentzian [a.k.a., Cauchy] profile)
  - Heisenberg uncertainty principle: $\Delta \nu = \Delta E / h$
- Collisional broadening (Lorentzian profile)
  - collisions interrupt photon emission process
  - $\Delta t_{\text{coll}} < \Delta t_{\text{emission}} \sim 10^{-9}$ s
    - dependent on $T$, $\rho$
- Pressure broadening ($\sim$ Lorentzian profile)
  - $\Delta t_{\text{interaction}} > \Delta t_{\text{emission}}$
  - nearby particles shift energy levels of emitting particle
    - Stark effect ($n = 2, 4$)
    - van der Waals force ($n = 6$)
    - dipole coupling between pairs of same species ($n = 3$)
    - dependent mostly on $\rho$, less on $T$
- Thermal Doppler broadening (Gaussian profile)
  - emitting particles have a Maxwellian distribution of velocities
- Rotational Doppler broadening (Gaussian profile)
  - radiation emitted from a spatially unresolved rotating body
- Composite line profile: Lorentzian + Gaussian = Voigt profile

$$I_\nu = I_0 \frac{\gamma / 2\pi}{(\nu - \nu_0)^2 + \gamma^2 / 4}$$

$$\gamma = \text{Lorentzian FWHM}$$

$$\gamma_{\text{natural}} = \frac{\Delta E_i + \Delta E_f}{h / 2\pi} = \frac{1}{\Delta t_i} + \frac{1}{\Delta t_f}$$

$$\gamma_{\text{collisional}} = 2/\langle \Delta t_{\text{coll}} \rangle$$

$$\gamma_{\text{pressure}} \propto \langle r \rangle^{-n}; \quad n = 2, 3, 4, 6$$

$$I_\nu = \frac{1}{\sqrt{2\pi} \Gamma} e^{-\frac{(\nu - \nu_0)^2}{2\Gamma^2}}$$

$$\Gamma = \text{Gaussian FWHM}$$

$$\Gamma_{\text{thermal}} = \nu_0 \sqrt{\frac{kT}{mc^2}}$$

$$\Gamma_{\text{rotational}} = 2\nu_0 u / c$$
Gravity-Sensitive Features in UCDs

(McGovern et al. 2004)
Gravity in UCDs

Key species:
- neutral alkali elements (Na, K)
  - weaker at low $g$
- hydrides
  - CaH weaker at low $g$
  - FeH unchanged
- oxides
  - VO, CO, TiO stronger at low $g$
  - H$_2$O $\sim$ unchanged

$log g$ and $T_{\text{eff}}$ are measurable properties

(Kirkpatrick et al. 2006)
Curve of Growth: Dependence of Line Equivalent Width $W$ on Column Density $N$

- $N \equiv$ integral of number density of absorbing atoms or molecules along line of sight [cm$^{-2}$]
  - for small $N$, $W \propto N$
    - linear part of the curve of growth
  - for larger $N$, $W \propto \sqrt{\ln N}$
    - after the Gaussian core bottoms out
    - flat part of the curve of growth
    - after the absorption by the Lorentzian wings becomes strong
    - square root part of the curve of growth
- There is a different curve of growth, $W(N)$, for each spectral line
Universal Curve of Growth

- the ratio of $W$ to Doppler line width $\Delta \lambda$ depends upon the product of $N$ and a line’s oscillator strength $f$ in the same way for every spectral line (e.g. Unsöld 1955).

\[
\log \left( \frac{W}{\Delta \lambda} \right) = \begin{cases} 
\text{linear} & W \propto N \\
\text{flat} & W \propto \sqrt{\ln N} \\
\text{square root} & W \propto \sqrt{N} 
\end{cases}
\]

\[
\Delta \lambda = \frac{\lambda}{c} \sqrt{2kT/m}
\]
Curve of Growth: Determining Abundances

• Measure $W$ for a lot of lines (each with distinct, known $f$) of a bunch of atomic or ionic species.

• Plot $W/\Delta \lambda$ against $xNf$ where:
  – $N$ is the column density of one species
  – $x$ is the **relative abundance** of the atomic species that gives rise to the line (ratio of number density of that species to the number density of the first species),

• Adjust $x$, $N$, and $\Delta \lambda$ until the points fit the universal curve of growth.

• Then one knows these three quantities for each species.
Subdwarf SEDs

Teff = 3500, logg = 5.0

Subdwarf SEDs with different metallicity:
- dM: $[m/H] = 0.0$
- esdM: $[m/H] = -2.0$
- usdM: $[m/H] = -4.0$

(Jao et al. 2008)
sdM’s
sdL’s
enhanced hydrides, H₂
Simple Chemical Picture of Atmospheric Cooling for MLT’s

• As gas temperature of a (brown) dwarf drops, atoms:
  – first favor an ionized state
    • e.g., Ca II, Fe II in Sun
  – then favor a neutral state
    • e.g., Na I, K I in M/L/T dwarfs
  – then form molecules
    • e.g, H₂O, TiO, FeH, CH₄ in M/L/T dwarfs
  – then condense into a solid or liquid
    • e.g., Mg₂SiO₄, Al₂O₃ in L/T dwarfs
    • dust clouds

• More refractory elements tend to condense first

• Exact sequence of molecule and condensate formation depends on
  – gas pressure
  – metallicity
  – turbulent mixing from warmer or colder layers, etc
Dust Cloud Chemistry

(Burrows et al. 2001)

FIG. 14. The logarithm (base ten) of the pressure (in atmospheres) vs the temperature (in K) for various brown dwarf models at 1 Gyr (in black, taken from Burrows et al., 1997) and for Jupiter. The yellow dots denote the positions of the photospheres, defined as where $T=T_{\text{eff}}$. Superposed on this figure are various condensation and composition transition lines, as well as cloud graphics indicating the approximate position of a cloud base. The green lines depict the $T/P$ trajectory for which the abundance of a neutral alkali atom equals that of its chloride, ignoring rainout(!). Also shown in blue are the enstatite, forsterite, and spinel condensation lines. In red on the left are the ammonia and water condensation lines and on the right are the iron and perovskite condensation lines. In dashed red are the CO/CH$_4$=1 and NH$_3$/N$_2$=1 lines. Inner adiabats for 0.08 M$_\odot$ and 0.09 M$_\odot$ models are also shown. Note that the cool upper reaches of an atmosphere are in the upper left [Color].
Cloud Level: Balance of Turbulent Mixing and Sedimentation

- Cloud condensates will settle under gravity to a level where there is enough upward convective (turbulent) motion to keep them afloat.
- Level and vertical extent of clouds depend on
  - droplet size (i.e., mass)
  - convective velocity, mixing efficiency

\[-K \frac{\partial q_t}{\partial z} - f_{\text{rain}} w_* q_c = 0, \quad K = \frac{H}{3} \left( \frac{L}{H} \right)^{4/3} \left( \frac{RF}{\mu \rho_a c_p} \right)^{1/3}\]

- $K$ – vertical eddy diffusion coefficient ($\sim 10^5$–$10^9$ cm$^2$ s$^{-1}$)
  - $H = RT/\mu g$ – atmospheric scale height ($\sim 10$ km); $L$ – turbulent mixing length ($\sim H$); $R$ – universal gas constant; $\mu$ – atmospheric molecular weight (2.2 g mol$^{-1}$ assumed); $\rho_a$ – atmospheric density; $c_p$ – specific heat of atmosphere at constant pressure (ideal gas); $F = \sigma T_{\text{eff}}^4$

- $q_c$ – condensate mixing ratio (mole of condensate per mole of atmosphere)
- $q_t = q_c + q_v$ – total mixing ratio (condensate + vapor)
- $w_* = K/L$ – convective velocity scale ($\sim 1$ m s$^{-1}$)
- $f_{\text{rain}}$ – sedimentation efficiency ($\sim 2$–$6$ in bulk of cumulus clouds on Earth)
  - ratio of mass-weighted droplet sedimentation velocity to $w_*$

(Ackerman & Marley 2001)
Condensate Clouds
(AM01 Baseline Models)

(Ackerman & Marley 2001)
Emergent Flux Depends on Wavelength and Cloud Level

\[ \tau_{\text{cloud}} < 0.5; \ h_{\text{cloud}} > h_{\text{photosphere}} \]

\[ \tau_{\text{cloud}} > 1; \ h_{\text{cloud}} \sim h_{\text{photosphere}} \]

\[ \tau_{\text{cloud}} > 1; \ h_{\text{cloud}} < h_{\text{photosphere}} \]

(Ackerman & Marley 2001)
Modeling L and T Dwarfs

- Models that incorporate suspended dust (DUSTY) successfully reproduce L dwarf colors

- Late T dwarfs well fit by dust-free photospheres (e.g., COND models: dust removed upon formation)

- Transition can be explained by sedimentation of silicate clouds below visible photosphere

(DUSTY models (dust remains suspended)

COND models (dust is removed)

--- 0.1 Gyr
--- 1 Gyr
--- 10 Gyr

\( M_K \) vs. \((J-K)\) (Baraffe et al. 2003)

Mar 30, 2009
The L/T Transition Problem

- photospheres turn blue in the near-IR unusually quickly
- clouds sink comparatively slowly
  - need to be “rained out” (sedimented) faster
- reddest L dwarfs require inefficient sedimentation ($f_{\text{rain}} < 3$)
- early T dwarfs require $f_{\text{rain}} > 3$
- late T’s require no visible clouds ($f_{\text{rain}} \rightarrow \infty$)
What Is the Weather on an Early T Dwarf?

• partly cloudy?

• uniformly hazy?

• raining “cats and dogs”?  
  – i.e., silicates and iron
Detecting Thermal Emission From Planet’s “Day” Side: Secondary Eclipse

Secondary Eclipse

See thermal radiation from planet disappear and reappear

Primary Eclipse

See radiation from star transmitted Through the planet’s atmosphere
Effect of Irradiation

- balance between internal flux and flux incident from star
  \[
  T_{\text{eff}}^4 = T_{\text{int}}^4 + W T^* T_{\text{int}}^4
  \]
- \( W \) – dimensionless “dilution” factor \( \sim 10^{-3} \)
- incident light penetrates to depth \( \tau_{\text{pen}} \), such that
  \[
  \tau_{\text{pen}} = W \left( \frac{T^*}{T_{\text{int}}} \right)^4 \approx 1
  \]
- for \( \tau < \tau_{\text{pen}} \), \( T_{\text{eff}} \) is governed by irradiation and is constant
  – isothermal, radiative region
- for \( \tau > \tau_{\text{pen}} \), \( T_{\text{eff}} \approx T_{\text{int}} \), and rises monotonically with \( \tau \)
P-T Profiles of Hot Jupiters

- isothermal regions are radiative

(Fortney et al. 2007)
Hot and Very Hot Jupiters: pL vs. pM Planets

- distinction:
  - based on lack or presence of high-level TiO/VO associated with a stratosphere
  - cf. L vs. M stellar spectral types
- transition at around 0.04–0.05 AU equivalent separation from the Sun
- note dependences on:
  - observed planetary hemisphere
  - orbital phase for planets on very eccentric orbits
    - HD 17156b, HD 80606b, HD 147506b

(Fortney et al. 2008)

Figure 1. Flux incident upon a collection of hot Jupiter planets, as of 30 June 2008. The labeled dotted lines indicate the distance from the Sun that a planet would have to be to intercept this same flux. Diamonds indicate the transiting planets while triangles indicate non-transiting systems (with minimum masses plotted). The error bars for some planets indicate the variation in flux that each receives over their eccentric orbits. Flux levels for pM Class and pL Class planets are shown, with the dark shaded region around ~0.04-0.05 AU indicating a possible transition region (based more on data available in Fall 2007, than on theory), from Fortney et al. (2008). The lighter shaded region reaches down to XO-1b.
Winds:
Cooling vs. Advection

- advection time scale
  \[ t_{\text{advec}} = \frac{R_p}{U} \]
  - \( R_p \) – planet radius
  - \( U \) – wind speed
- balance of cooling vs. advection decides wind speed \( U \)

\[
\frac{\Delta T_{\text{day-night}}}{\Delta T_{\text{rad}}} \sim 1 - e^{-t_{\text{advec}} / t_{\text{rad}}}
\]

- winds of several km/sec (~ sound speed) expected from 2D and 3D dynamical models
Radii of Very Hot Jupiters

- some large radii cannot be explained by coreless planet models with high-altitude stratospheres:
  - extra internal power source?
    - stratospheric heat trap
    - tidal heating
    - damping or orbital eccentricity and apparent resetting of planet age?
      - host stars are giga-years old
  - preferential evaporation of neutral helium?

(Fortney et al. 2007)
Rossiter-McLaughlin Effect

- first detected in eclipsing binary stars
  - as in bottom panel

- effective Doppler shift of (absorption) line changes depending on the part of the host star that is occulted

(Gaudi & Winn 2007)
RM Effect Geometry

• seek to measure angle $\lambda$ between projected stellar rotation axis and planetary orbital axis

(Ohta et al. 2005)

• note: $\Omega_S \sin I_S$ here is the same as $V_s \sin I_s$ and $V \sin I_s$ in the following slides: the projected stellar spin rate