Substellar Interiors

PHY 688, Lecture 13
Outline

• Review of previous lecture
  – curve of growth: dependence of absorption line strength on abundance
  – metallicity; subdwarfs

• Substellar interiors
  – equation of state (EOS)
  – thermodynamics of hydrogen
Previously in PHY 688...
Alkali (Na, K) lines in visible spectra of late-L and T dwarfs become saturated!
Lorentzian Line Profile at Increasing $\tau$

simulation for the H\textalpha line profile
Lorentzian Line Profile at Increasing $\tau$

simulation for the H$\alpha$ line profile

saturation at $\tau > 5$
Lorentzian Line Profile at Increasing $\tau$

Simulation for the H$\alpha$ line profile
Lorentzian vs. Gaussian Line Profiles: Small $\tau$

simulation for the H$\alpha$ line profile
Lorentzian vs. Gaussian Line Profiles: Large $\tau$

- simulation for the H$\alpha$ line profile
- core more sensitive to Gaussian parts
- wings more influenced by Lorentzian parts

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Universal Curve of Growth

- each absorbing species follows a different curve of growth
- however, ratio of $W$ to Doppler line width $\Delta \lambda$ depends upon the product of $N$ and a line’s oscillator strength $f$ in the same way for every spectral line
- useful for determining element abundances

$$\log \left( \frac{W}{\Delta \lambda} \right)$$

- linear: $W \propto N$
- flat: $W \propto \sqrt{\ln N}$
- square root: $W \propto \sqrt{N}$

$$\Delta \lambda = \frac{\lambda}{c} \sqrt{\frac{2kT}{m}}$$
Metallicity

\([m/H]_* = \log N(m)_*/ \log N(H)_* - \log N(m)_{\text{Sun}}/ \log N(H)_{\text{Sun}}\)

- \([\text{Fe/H}]_{\text{Sun}} = 0.0\) is the solar Fe abundance
- \([\text{Fe/H}] = -1.0\) is the same as 1/10 solar

\([m/\text{Fe}]_* = \log N(m)_*/ \log N(\text{Fe})_* - \log N(m)_{\text{Sun}}/ \log N(\text{Fe})_{\text{Sun}}\)

- \([\text{Ca/Fe}] = +0.3\) means twice the number of Ca atoms per Fe atom

- Sun’s metal content is \(Z \approx 1.6\%\) by mass
The Early Universe Had No Heavy Elements

(figure credit: N. Wright, UCLA)
Metal Enrichment Is Due to Stars

Diagram showing the abundance of elements plotted against atomic number, with Li, Be, B, C, O, Mg, Si, S, Fe, and others, and red arrows indicating nuclear fusion and neutron capture.
Stellar Populations

- **Young stars (Pop I) are metal-rich**
  - \([m/H] > -2.0\)
  - Milky Way disk, spiral arms

- **Old stars (Pop II) are metal-poor**
  - \([m/H] < -2.0\)
  - Milky Way halo, bulge

- **Ancient stars (Pop III) are expected to have been nearly metal-free (at birth)**
Cool Metal-Poor Stars

- M-type metal-poor stars are most numerous
  - M (+L) stars are longest lived
- $dM$: normal M dwarfs
  $[m/H] > -1$
- $sdM$: M subdwarfs
  $\langle [m/H] \rangle \sim -1.3$
- $esdM$: M extreme subdwarfs
  $\langle [m/H] \rangle \sim -2$
  - i.e., Pop II
- $usdM$: M ultra subdwarfs
  $\langle [m/H] \rangle < -2$
  - also Pop II
Cool Metal-Poor Stars

- lower atmospheric opacity of subdwarfs reveals the deeper, hotter layers of these stars
- a dM and an sdM star of the same mass have approximately the same luminosity, but the sdM star can be up to 700 K hotter
- subdwarfs : dwarfs ~ 1 : 400 (in Milky Way)
- sd’s identified by their high velocities relative to Sun
  - kinematics characteristic of galactic halo stars
Subdwarf SEDs

- signatures of metal deficiencies
- higher gravity in deeper layers?

$dM_{esdM_{usdM}}$

(Jao et al. 2008)
sdM’s + sdL’s

• enhanced hydrides, CIA H₂

(Burgasser 2008)
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0.05 $M_{\odot}$ Brown dwarf at 5 Gyr

($X = 100\%$ H)

(1 bar = $10^6$ dyn/cm$^2 = 0.99$ atm)
From Lecture 4: Why Are Low-Mass Stars and Brown Dwarfs Fully Convective?

- **radiation**
  - photons absorbed by cooler outer layers
  - efficient in:
    - $>1 \ M_{\text{Sun}}$ star envelopes
    - cores of 0.3–1 $M_{\text{Sun}}$ stars
    - all stellar photospheres

\[
\left( \frac{dT}{dr} \right)_{\text{rad}} = -\frac{3\kappa \rho L_r}{64 \pi r^2 \sigma T^3}
\]

- **convection**
  - adiabatic exponent $\gamma = C_P/C_V$
  - important when radiation inefficient:
    - interiors of brown dwarfs and $<0.3 \ M_{\text{Sun}}$ stars
    - cores of $>1 \ M_{\text{Sun}}$ stars
    - envelopes of $\sim 1 \ M_{\text{Sun}}$ stars

\[
\left( \frac{dT}{dr} \right)_{\text{ad}} = \left( 1 - \frac{1}{\gamma} \right) \frac{T}{P} \frac{dP}{dr}
\]

- **in low-mass stars and brown dwarfs:**
  - large opacity $\kappa \Rightarrow (dT/dr)_{\text{rad}}$ steepens
  - more d.o.f. per gas constituent

\[n = \text{d.o.f} / 2; \quad \gamma = 1 + 1/n \Rightarrow (dT/dr)_{\text{ad}} \text{ becomes shallower}\]
Convective Interiors Mean That:

• entropy \((S)\) is constant throughout
  \(- dS = dQ / T = 0\) in convective interior (disregarding radiative atmosphere)

• equation of state is adiabatic
  \(- \Delta S = 0\) holds for a reversible adiabatic process

• \(P = K\rho^\gamma\) \((\gamma = 1+1/n, n = 1.5)\)

• brown dwarfs are polytropes of index \(n = 1.5\)
From Lecture 4: A Special Solution to the EOS – Stars as Polytropes

• \( P \equiv P(\rho) = K\rho^\gamma, \quad \gamma = 1 + 1/n \)
  
  \( K \) - constant, \( n \) - polytropic index

• Lane-Emden equations:
  
  – dimensionless forms of equation of hydrostatic equilibrium
  
  – solutions:
  \[
P = P_0\theta^{n+1}, \quad \rho = \rho_0\theta^n, \quad T = T_0\theta
\]

• important polytropes:
  
  – \( n = 3 \): normal stars
  
  – \( n = 1.5 \): brown dwarfs, planets, white dwarfs (all are degenerate objects)
  
  – \( n = 1 \): neutron stars
  
  – \( n = \infty \): isothermal proto-stellar clouds

\[
\frac{1}{\xi^2} \frac{d}{d\xi} \left( \xi^2 \frac{d\xi}{d\theta} \right) + \theta^n (\xi) = 0
\]

\[
\frac{\xi}{a} = \frac{r}{a}, \quad a = \left[ (n + 1) \frac{K}{4\pi G \rho_c^{(1-n)/n}} \right]^{1/2}
\]

\( \theta \) – dimensionless potential

Boundary conditions:

\( \theta(\xi = \xi_1) = 0 \)  stellar surface

\( \theta(\xi = 0) = 1 \)  stellar core

\[
\left( \frac{d\theta}{d\xi} \right)_{\xi=0} = 0 \quad \text{stellar core}
\]
Solutions for Substellar Polytropes

\[ \rho_c \propto M^{2n/(3-n)} \]
\[ P_c \propto M^{2(1+n)/(3-n)} \]
\[ R \propto M^{(1-n)/(3-n)} \]  \hspace{1cm} (Burrows & Liebert 1993)

- \( n = 1.5 \): \( R \propto M^{-1/3} \)
- \( n = 1.0 \): \( R \propto M^0 = \text{const} \)
  - important for \( M < 4 M_{\text{Jup}} \) objects, in which there are Coulomb corrections to \( P(\rho) \) degenerate EOS
- analytic fit for brown dwarfs and planets (Zapolsky & Salpeter 1969)

\[ R = 2.2 \times 10^9 \left( \frac{M_{\text{Sun}}}{M} \right)^{1/3} \left[ 1 + \left( \frac{M}{0.0032M_{\text{Sun}}} \right)^{-1/2} \right]^{4/3} \text{ cm} \]
Substellar radius changes by <50%; always near 1 $R_{\text{Jup}} \approx 0.12 R_{\text{Sun}}$

(Burrows & Liebert 1993)
EOS and Thermodynamics of Hydrogen

- 10 orders of magnitude change in pressure
- 3 orders of magnitude change in temperature
- expect different phases of hydrogen

0.05 M⊙ Brown dwarf at 5 Gyr
(X = 100% H)

P ≈ 5 bar, T ≈ 1000 K, ρ ≈ 10⁻⁴ g/cc

atmosphere (~10 km)

H₂ fluid

phase change?

Metallic H

P ≈ 10¹¹ bar, T ≈ 10⁸ K, ρ ≈ 500 g/cc
Hydrogen phase diagram

(Burrows & Liebert 1993)
Low-Density Regime

• temperature is sufficient to excite rotational levels of $\text{H}_2$ into equipartition, although not the vibrational levels
  – d.o.f. of $\text{H}_2$ molecules:
    • 3 (spatial motion) + 2 (rotation around short axes) = 5
  – polytropic index $n = \text{d.o.f.}/2 = 2.5 \Rightarrow \gamma = 1+1/n = 1.4$
  – $P = K\rho^\gamma; P = \rho kT/\mu \Rightarrow T \propto \rho^{\gamma-1} = \rho^{0.4}$

• compare with $T \propto \rho^{0.67}$ for $n = 1.5$ polytrope, as in:
  – high-density interiors of brown dwarfs
  – Jupiter, which does not have sufficient temperature in atmosphere to excite $\text{H}_2$ rotations into equipartition
Hydrogen phase diagram

\[ T \propto \rho^{0.4} \]

\[ T \propto \rho^{0.67} \]

(Burrows & Liebert 1993)
High-Density Regime

• expect phase change at low $T$, sufficiently high $\rho$, $P$
  – plasma phase transition (PPT)
  – pressure ionization and metallization of H or H+He mixture
  – occurs at $\rho \sim 1 \text{ gm/cm}^3$, $P = 1$–$3 \text{ Mbar}$

• interior is “strongly coupled Coulomb plasma”
  – bulk of Jupiter ($\sim 85\%$), Saturn ($\sim 50\%$), brown dwarfs ($>99.9\%$)
Onset of Coulomb Dominance

- $\Gamma$ – (dimensionless) ratio of Coulomb energy per ion to $kT$
- $Ze$ – ionic charge ($Z=1$ for H)
- $T_6 = T / 10^6$ K
- $\mu_e$ – number of baryons per electron
- $n_e = \rho N_A \mu_e$ – electron density
- relevant length scales
  - $r_s$ – radius of sphere that contains one nucleus on average
  - $r_e$ – electron spacing parameter
- $\Gamma \ll 1$ : weekly coupled regime (Debye-Hückel limit)
  - Sun, blood plasma
- $\Gamma \sim 1$ : transition to strong coupling
  - brown dwarf matter, planetary interiors : $1 < \Gamma < 50$

$$\Gamma = \frac{Z^2 e^2}{r_s kT} = 0.227 \left( \frac{\rho}{\mu_e} \right)^{1/3} Z^{5/3} / T_6$$

$$\frac{1}{\mu_e} = X + \frac{1}{2} Y = \frac{1+X}{2}$$

$$r_s = \left( \frac{3Z}{4\pi n_e} \right)^{1/3} = \left( \frac{3\mu_e Z}{4\pi N_A \rho} \right)^{1/3}$$

$$r_e = \left( \frac{3}{4\pi n_e} \right)^{1/3} = \left( \frac{3\mu_e}{4\pi N_A \rho} \right)^{1/3} = \frac{r_s}{Z^{1/3}}$$
Hydrogen phase diagram

(Burrows & Liebert 1993)