1. (20 points) The equation of state of planets near the mass of Jupiter can be fitted by an $n = 1$ polytrope, $p = K \rho^2$, with $K$, a physical constant. Show that the equation of hydrostatic equilibrium and Poisson’s equation applied to such a planet lead to

$$\nabla^2 \rho = -\frac{2\pi G}{K} \rho.$$

a. Find $\rho(r)$ for a spherical planet with this equation of state. How does its radius depend on $K$ and on the central density of the planet.

b. Show that the equation also admits a separated solution for a rectangular planet centered at the origin:

$$\rho = \rho_0 \cos \frac{\pi x}{L_x} \cos \frac{\pi y}{L_y} \cos \frac{\pi z}{L_z}, \quad |x| < \frac{L_x}{2}, |y| < \frac{L_y}{2}, |z| < \frac{L_z}{2}.$$ 

What conditions, if any, need to be satisfied?

c. Could these rectangular Jupiters exist? Justify your answer in mathematical and physical detail.

2. (10 points) Exercise 10.1 of HKT. Note that the exercise mistakenly refers to Figure 10.4, rather than 10.1.