Statistics, Error Analysis
Hypothesis Testing

PHY517 / AST443, Lecture 5
Remote Login Issues

- Need an Xserver to display graphics remotely
- Instructions on how to install one for Windows, Mac OS are now available on course website
- Ask for a no-penalty extension if this slowed you down
Outline

• Statistics
  – statistical distributions
  – expectations, error analysis
  – signal-to-noise estimation

• Hypothesis testing
  – parametric tests: t test, F test,
  – non-parametric tests: $\chi^2$ test, K-S test
Basic Concepts

- Binomial, Poisson, Gaussian distributions
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• Binomial, Poisson, Gaussian distributions

• probability density function (p.d.f.)
  – density of probability at each point
  – probability of a random variable falling within a given interval is the integral over the interval
Basic Concepts

• Central Limit Theorem:
  “Let $X_1, X_2, X_3, \ldots, X_n$ be a sequence of $n$ independent and identically distributed random variables each having finite expectation $\mu > 0$ and variance $\sigma^2 > 0$. As $n$ increases, the distribution of the sample average approaches the normal distribution with a mean $\mu$ and variance $\sigma^2 / n$ irrespective of the shape of the original distribution.”
Demonstration of Central Limit Theorem

A bizarre p.d.f. \( p(x) \) with \( \mu = 0, \sigma^2 = 1 \)

p.d.f. of sum of 2 random variables sampled from \( p(x) \)
(i.e., autoconvolution of \( p(x) \))

p.d.f. of sum of 3 random variables sampled from \( p(x) \)

p.d.f. of sum of 4 random variables sampled from \( p(x) \)

source: wikipedia
Confidence Intervals

\[ 1 - \alpha = 0.95 \]
\[ \alpha = 0.05 \]

\[ z_{CRIT} = 1.65 \]

\[ 1 - \alpha = 0.99 \]
\[ \alpha = 0.01 \]

\[ z_{CRIT} = 2.33 \]
Types of Error in Hypothesis Testing
Student’s $t$ Distribution

$k = \text{d.o.f.}$

source: wikipedia
F Distribution

\[ \text{d1, d2 = d.o.f.} \]
$\chi^2$ Distribution

(Wall & Jenkins 2008; Fig 5.4)