Administrative

- **Homework 2:**
  - problems 4.4, 5.1, 5.3, 5.4 of W&J
  - due date extended until Friday, Oct 23
    - drop off in my office before 6pm

- **Midterm: Monday, Oct 26**
  - 8:20–9:20pm in ESS 450
  - closed book
  - no cheat sheet
  - no need to memorize obscure constants or formulae
Midterm Review

- Coordinates and time
- Detection of light:
  - telescopes, detectors
- Radiation:
  - specific intensity, flux density, etc
  - optical depth, extinction, reddening
  - differential refraction
- Magnitudes and photometry:
  - apparent, absolute; distance modulus
  - CMDs and CCDs
  - photometry, PSF fitting
- Statistics
  - testing correlations and hypotheses
  - non-parametric vs. parametric tests
  - one-tailed vs. two-tailed tests
  - one-sample, two-sample tests
Celestial Coordinates

• **horizon coordinates**
  – altitude/elevation \((a)\), azimuth \((A)\)
  – zenith, zenith angle \((z)\)

• observer’s latitude
  – angle PZ
Celestial Coordinates

- **equatorial coordinates**
  - right ascension
    - R.A., $\alpha$
  - declination
    - DEC, $\delta$
- cf., Earth’s longitude, latitude
- meridian, hour angle ($HA$)
Celestial Coordinates

- **ecliptic coordinates**
  - ecliptic longitude ($\lambda$)
  - ecliptic latitude ($\beta$)

- vernal equinox ($\Upsilon$)
  - Earth passes through ecliptic on Mar 20/21
  - origin of equatorial and ecliptic longitude

- Earth’s axial tilt:
  - $\varepsilon = 23.439281^\circ$
  - a.k.a., “obliquity of the ecliptic”
Celestial Coordinates

• **galactic coordinates**
  – galactic longitude ($l$; letter “ell”)
    • $l = 0$ approx. toward galactic center (GC)
    • definition
      $l_{\text{NCP}} = 123^\circ$ (B1950.0)
  – galactic latitude ($b$)
    • NGP definition (B1950.0)
      $\alpha = 12h\ 49m$
      $\delta = 27.4^\circ$
    • Sagittarius A
      $l = 359^\circ\ 56'\ 39.5''$
      $b = -0^\circ\ 2'\ 46.3''$
Coordinate Transformations

- **equatorial ↔ ecliptic**
  \[
  \cos \delta \cos \alpha = \cos \beta \cos \lambda \\
  \cos \delta \sin \alpha = \cos \beta \sin \lambda \cos \varepsilon - \sin \beta \sin \varepsilon \\
  \sin \delta = \cos \beta \sin \lambda \sin \varepsilon + \sin \beta \cos \varepsilon \\
  \cos \beta \sin \lambda = \cos \delta \sin \alpha \cos \varepsilon + \sin \delta \sin \varepsilon \\
  \sin \beta = \sin \delta \cos \varepsilon - \cos \delta \sin \alpha \sin \varepsilon
  \]

- **equatorial ↔ horizontal**
  \[
  \cos a \sin A = -\cos \delta \sin HA \\
  \cos a \cos A = \sin \delta \cos \phi - \cos \delta \cos HA \sin \phi \\
  \sin a = \sin \delta \sin \phi + \cos \delta \cos HA \cos \phi \\
  \cos \delta \sin HA = -\cos a \sin A \\
  \sin \delta = \sin a \sin \phi + \cos a \cos A \cos \phi
  \]

\[\phi \equiv \text{observer's latitude}\]
Equatorial Coordinate Systems

- **FK4**
  - precise positions and motions of 3522 stars
  - adopted in 1976
  - B1950.0
- **FK5**
  - more accurate positions
  - fainter stars
  - J2000.0
- **ICRS (International Celestial Reference System)**
  - extremely accurate (± 0.5 milli-arcsec)
  - 250 extragalactic radio sources
    - negligible proper motions
  - J2000.0
Astronomical Time

- sidereal time
  - determined w.r.t. stars
  - local sidereal time (LST)
    - R.A. of meridian
    - HA of vernal equinox
  - sidereal day: 23h 56m 4.1s
- object's hour angle
  \[ HA = LST - \alpha \]
Astronomical Time

• **sidereal time**
  – determined w.r.t. stars
  – local sidereal time (LST)
    • R.A. of meridian
    • $HA$ of vernal equinox
  – sidereal day: 23h 56m 4.1s

• **object’s hour angle**
  $HA = LST - \alpha$

• **solar time**
  – solar day is 3 min 56 sec longer than sidereal day
Astronomical Time

• universal time
  – UT0: determined from celestial objects
    • corrected to duration of mean solar day
    • HA of the mean Sun at Greenwich (a.k.a., GMT)
  – UT1: corrected from UT0 for Earth’s polar motion
    • 1 day = 86400 s, but duration of 1 s is variable
  – UTC: atomic timescale that approximates UT1
    • kept within 0.9 sec of UT1 with leap seconds
    • international standard for civil time
    • set to agree with UT1 in 1958.0
Astronomical Time

- tropical year
  - measured between successive passages of the Sun through the vernal equinox
  - 1 yr = 365.2422 mean solar days
- mean sidereal year
  - Earth: 50.3"/yr precession in direction opposite of solar motion
  - 365.2564 days
- Julian calendar
  - leap days every 4th year; 1 yr = 365.25 days
  - $t_0 = $ noon on Jan 1st, 4713 BC
- Gregorian calendar
  - no leap day in century years not divisible by 400 (e.g., 1900)
  - 1 yr = 365.2425 days
Coordinate Epochs

• Coordinates are given at \textit{B1950.0} or \textit{J2000.0 epochs}
  – Besselian years (on Gregorian calendar; tropical years)
  – Julian years (Julian calendar)

• Gregorian calendar is irregular
  – complex for precise measurements over long time periods

• Julian epoch:
  – Julian date: \( JD = 0 \) at noon on Jan 1, 4713 BC
  – \( J = 2000.0 + (JD - 2451545.0) / 365.25 \)
  – \( J2000.0 \) defined at
    • JD 2451545.0
    • January 1, 2000, noon
Focusing

• focal length \((f_L)\), focal plane

• object size \((\alpha, s)\) in the focal plane
  \[ s = f_L \tan \alpha \approx f_L \alpha \]

• plate/pixel scale
  \[ P = \alpha / s = 1 / f_L \]
  – Lick observatory 3m
    • \(f_L = 15.2\)m, \(P = 14''/mm\)
Energy and Focal Ratio

• Specific intensity:  
  - Planck law  
  - \([\text{erg s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1} \text{ sterad}^{-1}] \) or \([\text{Jy sterad}^{-1}]\)

• Integrated apparent brightness

\[ E_p \propto \left( \frac{d}{f_L} \right)^2 : \text{energy per unit detector area} \]

• focal ratio: \( R = \frac{f_L}{d} \)
  - “fast” \((< f/3)\) vs. “slow” optics \((> f/10)\)
  - fast data collection vs. larger magnification

  \[ \text{magnification} = \frac{f_L}{f_{\text{camera}}} \]
Optical Telescope Architectures

Also:

- **Schmidt-Cassegrain**
  - spherical primary (sph. aberration), corrector plate; cheap for large FOV
  - no coma or astigmatism; severe field distortion

- **Ritchey-Chrétien**
  - modified Cassegrain with hyperbolic primary and hyperbolic convex secondary
  - no coma; but astigmatism, some field distortion
Imaging through a Turbulent Atmosphere: Seeing

- FWHM of seeing disk
  - $\theta_{\text{seeing}} < 1.0''$ at a good site
- $r_0$: Fried parameter
  - $\theta_{\text{seeing}} = 1.2 \, \lambda / r_0$
  - $r_0 \propto \lambda^{6/5} (\cos z)^{3/5}$
  - $\theta_{\text{seeing}} \propto \lambda^{-1/5}$
- $t_0$: coherence time
  - $t_0 = r_0 / v_{\text{wind}}$
  - $v_{\text{wind}} \sim$ several m/s
  - $t_0$ is tens of milli-sec
Basic Concept of a Semi-Conductor Detector

- electron-hole pair generation
- doping:
  - n-type (electrons)
  - p-type (holes)
  - creates additional energy levels within band gap
  - *increases conductivity*
- silicon (Si)
  - band gap: $E_g = 1.12 \text{ eV}$
    - cut-off wavelength $\lambda_c = 1.13 \mu\text{m}$
    \[
    \lambda_c = \frac{hc}{E_g} = \frac{1.24 \mu\text{m}}{E_g (\text{eV})}
    \]
  - free-electron energy: 4 eV (3000Å)
  - 1 photon -> 1 electron
Basic Concept of a CCD Pixel: A P-N Photo Diode

- depleted region
  - low conductivity
  - can support an E field
- net positive charge (higher charge density near top)
- additional E-field applied
- subsequently generated electrons get trapped in potential well near top
Figure 2b  The basic layout of a three-phase two-dimensional CCD. The sequence 1, 2, 3 on each set of electrodes indicates the normal direction of charge transfer in the parallel and serial registers.
Front- vs. Back-Illumination

Figure 2

Frontside and Backside Illuminated CCDs

Figure 3

Frontside and Backside CCD Quantum Efficiency

- UV Coating
- Back Illuminated CCD
- Front Illuminated CCD
Read Noise

- electrons / pix / read
- sources
  - A/D conversion not perfectly repeatable
  - spurious electrons from electronics (e.g., from amplifier heating)
- alleviated through cooling
- nowadays: <3–10 electrons
Dark Current

• electrons / pixel / second
• source
  – thermal noise at non-zero detector temperature
• higher at room temperature (~10,000)
• at cryogenic temperatures
  – LN2, –100 C
  – 0.1–20 e⁻/pix/s
Detector Calibration

• bias frames
  – non-zero bias voltage
  – 0s integrations

• dark frames
  – equal to science integrations

• flat field frames
  – QE of detector pixels is non-uniform in 2-D
  – QE is dependent on observing wavelength

• bad pixels
Sky and Telescope Calibration

- sky background images
- photometric calibration:
  - airmass curve, filter transmission
- sky transmission
  - spectrum of a star of a known spectral energy distribution
- astrometric calibration
  - binary with a known orbit
  - star-rich field with precisely known positions: HST observations of globular clusters
- point-spread function calibration
  - nearby bright star (for on-axis calibration)
  - star-rich field (2-d information on PSF)
Aperture Photometry

• object flux = total counts – sky counts
• estimation of background
  – $N_{\text{pix, bkg}} > 3 N_{\text{pix, src}}$
  – use $r_{\text{bkg}} >> \text{FWHM}$, whenever possible
• enclosed energy $P(r)$
  – “curve of growth”
• optimum aperture radius $r$
  – $\text{SNR}(r)$ first increases, then decreases with $r$
    • Fig. 5.7 of Howell
  – dependent on PSF FWHM and source brightness
Transmission vs. air mass

Transmission

Extinction per air mass

source: Kitt Peak National Observatory
Radiation

• specific intensity $I_\nu$
  – $dE = I_\nu \, dt \, dA \, d\nu \, d\Omega$ [erg s$^{-1}$ cm$^{-2}$ Hz$^{-1}$ sterad$^{-1}$] or [Jy sterad$^{-1}$]
  – 1 Jy = 10$^{-23}$ erg s$^{-1}$ cm$^{-2}$ Hz$^{-1}$ = 10$^{-26}$ W m$^{-2}$ Hz$^{-1}$
  – surface brightness of extended sources (independent of distance)

• spectral flux density $S_\nu$
  – $S_\nu = fI_\nu \, d\Omega$ [erg s$^{-1}$ cm$^{-2}$ Hz$^{-1}$] or [Jy] or [W m$^{-2}$ Hz$^{-1}$]
  – point sources, integrated light from extended sources

• flux density $F$
  – $F = fS_\nu \, d\nu$ [erg s$^{-1}$ cm$^{-2}$] or [W m$^{-2}$]

• power $P$
  – $P = fF \, dA = \frac{dE}{dt}$ [erg s$^{-1}$] or [W]
  – received power: integrated over telescope area
  – luminosity: integrated over area of star

• conversion to photon counts
  – energy of $N$ photons: $Nh\nu$
Blackbody Radiation (Lecture 4)

- **Planck law**
  - specific intensity

- **Wien displacement law**
  \[ T \lambda_{\text{max}} = 0.29 \text{ K cm} \]

- **Stefan-Boltzmann law**
  \[ F = \sigma T^4 \]
  \[ \sigma = \frac{2\pi^5 k^4}{15c^2h^3} = 5.67 \times 10^{-5} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ K}^{-4} \]

- **Stellar luminosity**
  - power
  - [erg s\(^{-1}\)]

- **Inverse-square law**
  \[ L(r) = L_\star \frac{I(r)}{r^2} = 4\pi R_\star^2 \sigma T_{\text{eff}}^4 \]
Blackbody Radiation (Lecture 4)

\[ T_{\text{eff, Sun}} = 5777 \text{ K} \]
\[ T \lambda_{\text{max}} = 0.29 \text{ K cm} \]
Magnitudes (**Lecture 4**)

- **apparent magnitude:**  \( m = -2.5 \log \frac{F}{F_0} \)
  - \( m \) increases for fainter objects!
  - \( m = 0 \) for Vega; \( m \sim 6 \) mag for faintest naked-eye stars
  - faintest galaxies seen with Hubble: \( m \approx 30 \) mag
    - \( 10^{9.5} \) times fainter than faintest naked-eye stars
  - **dependent on observing wavelength**
    - \( m_V, m_B, m_J, \) or simply \( V \) (550 nm), \( B \) (445 nm), \( J \) (1220 nm), etc

- **bolometric magnitude (or luminosity):** \( m_{\text{bol}} \) (or \( L_{\text{bol}} \))
  - normalized over all wavelengths
Magnitudes and Colors

(Lecture 4)

• magnitude differences:
  – relative brightness
    \[ V_1 - V_2 = -2.5 \log \frac{F_{V1}}{F_{V2}} \]
  • \( \Delta m = 5 \) mag approx. equivalent to \( F_1/F_2 = 100 \)
  – color
    \[ B - V = -2.5 \left( \log \frac{F_{B}}{F_{V}} - \log \frac{F_{B,Vega}}{F_{V,Vega}} \right) \]
Extinction and Optical Depth
(Lecture 4)

- Light passing through a medium can be:
  - transmitted, absorbed, scattered
- **extinction** at frequency $\nu$ over distance $s$
  $$dL_{\nu}(s) = -\kappa_{\nu} \rho L_{\nu} \, ds = -L \, d\tau_{\nu}$$
  $$L_{\nu} = L_{\nu,0} e^{-\tau} = L_{\nu,0} e^{-\kappa \rho s} = L_{\nu,0} e^{-s/l}$$
  $$A_{\nu} = 2.5 \lg (F_{\nu,0}/F_{\nu}) = 2.5 \lg(e) \tau_{\nu} = 0.43 \tau_{\nu} \text{ mag}$$
  - medium opacity $\kappa_{\nu}$ [cm$^2$ g$^{-1}$], density $\rho$ [g cm$^{-3}$]
  - optical depth $\tau_{\nu} = \kappa_{\nu} \rho s$ [unitless]
  - photon mean free path: $l_{\nu} = (\kappa_{\nu} \rho)^{-1} = s/\tau_{\nu}$ [cm]
  $$A_V = m_V - m_{V,0}$$
- **reddening** between two frequencies ($\nu_1$, $\nu_2$)
  $$E_{\nu_1,\nu_2} = m_{\nu_1} - m_{\nu_2} - (m_{\nu_1} - m_{\nu_2})_0 \quad \text{[mag]}$$
  - $(m_{\nu_1} - m_{\nu_2})_0$ is the intrinsic color of the star
Interstellar Extinction Law

Figure 3.17 The interstellar extinction curve for two representative lines of sight. The full curve is characteristic of lines of sight that pass only through the intercloud medium, while the dashed curve is for a line of sight that penetrates deep into a molecular cloud. [From data published in Mathis (1990)]

extinction is highest at ~100 nm = 0.1 µm
unimportant for >10 µm
Interstellar Extinction Law

- \( A_V / E(B-V) = 3.1 \)
  - \( A_V / E(J-K) = 5.8 \)
  - \( A_V / E(V-K) = 1.13 \)
  - \( A_\lambda / E(J-K) = 2.4 \lambda^{-1.75} \) (0.9 < \( \lambda < 6 \mu m \))

- \( A_V \approx 0.6 r / (1000 \text{ ly}) \text{ mag} \)
  - \( b < 2^\circ \) (galactic latitude)

- \( A_V \approx 0.18 / \sin b \) mag
  - \( b > 10^\circ \)

- \( N_H / A_V \approx 1.8 \times 10^{21} \text{ atoms cm}^{-2} \text{ mag}^{-1} \)
  - atoms of neutral hydrogen (H I)
The Bessell approximations to UBVRI passbands

Overall transmission

Wavelength (Angstroms)
Atmospheric Transmission and Near-IR Filters
Extinction and Reddening: CMD

- Legend:
  - arrow: $A_V = 5 \text{ mag}$ extinction
  - solid line: main sequence
  - dotted line: substellar models
  - crosses: known brown dwarfs
  - solid points: brown dwarf candidates

Metchev et al. (2003)
Extinction and Reddening: CCD

- Legend:
  - arrow: $A_V = 5$ mag extinction
  - solid line: main sequence + giants
  - dotted line: substellar models
  - crosses: known brown dwarfs
  - solid points: brown dwarf candidates

Metchev et al. (2003)
higher ionization potential species
Statistics: Basic Concepts

• Binomial, Poisson, Gaussian distributions
  – calculating means and variances

• Central Limit Theorem:
  “Let $X_1, X_2, X_3, \ldots, X_n$ be a sequence of $n$ independent and identically distributed random variables each having finite expectation $\mu > 0$ and variance $\sigma^2 > 0$. As $n$ increases, the distribution of the sample average approaches the normal distribution with a mean $\mu$ and variance $\sigma^2 / n$ irrespective of the shape of the original distribution.”
A bizarre p.d.f. $p(x)$ with $\mu = 0$, $\sigma^2 = 1$

p.d.f. of sum of 2 random variables sampled from $p(x)$ (i.e., autoconvolution of $p(x)$)

p.d.f. of sum of 3 random variables sampled from $p(x)$

p.d.f. of sum of 4 random variables sampled from $p(x)$

source: wikipedia
Statistics: Basic Concepts

- probability density function (p.d.f.)
  - density of probability at each point
  - probability of a random variable falling within a given interval is the integral over the interval
Fig. 9. The Formulation of the Velocity-Distance Relation.
Student’s $t$ Distribution

$k = \text{d.o.f.}$

source: wikipedia
F Distribution

d1, d2 = d.o.f.
\[ \chi^2 \text{ Distribution} \]

\[ f(\chi^2, k) \]

source: wikipedia
\( \chi^2 \) Distribution

- probability that measured \( \chi^2 \) or higher occurs by chance under \( H_0 \)

source: wikipedia