Astrometry

AST443, Lecture 15
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Administrative

• Reading:
  – Howell, chapters 5.8

• Project 4, part II:
  – you now have positions and instrumental magnitudes for all your stars
  – calibrate your photometry
    • relative to the standard stars (open clusters)
    • relative to the mean change in magnitude of field stars (transiting planet)
  – begin data analysis
    • plot CMDs and CCDs; separate known cluster members (open clusters)
    • eliminate variable stars; plot time variation of observed flux (transiting planet)
  – complete by Mon, Nov 23
Outline

• Reference system
• Apparent stellar motions due to Earth’s orbital motion and axial rotation
• Proper stellar motion
• Measuring positions and motions
  – tangent-plane geometry
  – PSF sampling
  – focal plane distortion
• Example: the HR 8799 triple planetary system
Reference (from Lecture 3): Equatorial Coordinate Systems

- **FK4**
  - precise positions and motions of 3522 stars
  - adopted in 1976
  - B1950.0

- **FK5**
  - more accurate positions (± 50 milli-arcsec), fainter stars
  - adopted in 1988
  - J2000.0

- **ICRS (International Celestial Reference System)**
  - extremely accurate (± 0.5 milli-arcsec)
  - adopted in 1998
  - 250 extragalactic radio sources
    - Very-Long Baseline Interferometry (VLBI) measurements
    - negligible proper motions
  - J2000.0
Reference Point

• first point of Aries
  – a.k.a., “vernal equinox”
  – R.A. = 0, DEC = 0
  – $\lambda = 0, \beta = 0$

• this is a fictitious, moving point!
Precession and Nutation
(from Lecture 3)

• The Earth **precesses**…
  – Sun’s and Moon’s tidal forces
  – precession cycle: 25,800 years
  – rate is 1° in 72 years (along precession circle) = 50.3″/year

• … and **nutates**
  – Sun and Moon change relative locations
  – largest component has period of 18.6 years (19″ amplitude)
Precession

• Vernal equinox follows a retrograde motion
  – 25725 year cycle (Platonic year)
  – ecliptic coordinates increase monotonically
• In ecliptic coordinates:
  \[
  P_0 = 50.3878'' + 0.000049'' \, T \\
  P_1 = -0.1055'' + 0.000189'' \, T \\
  P = P_0 + P_1 \cos \varepsilon = 50.2910'' + 0.000222'' \, T
  \]
  \[T = \text{number of tropical years since 1900}
  \]
  \[\varepsilon = 23.439281^\circ \text{ (obliquity of the ecliptic = Earth’s axial tilt)}\]
• In equatorial coordinates
  \[
  m = 46.124'' + 0.000279'' \, T \\
  n = 20.043'' + 0.000085'' \, T \\
  P_\alpha = m + n \sin \alpha \tan \delta \\
  P_\delta = n \cos \alpha
  \]
• Nutation: periodic change in rate of precession
  – e.g., NCP traces a 9.2'' ellipse every 18.6 yr due to inclination of Moon’s orbit
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Annual Stellar Aberration

• Earth’s orbital motion causes a period aberration in an object’s position
  – velocity of light is finite
  – angular size of correction is $\sim v/c$

\[
\begin{align*}
\Delta \lambda &= -k \cos(\lambda - \lambda_0) \sec \beta \\
\Delta \beta &= -k \cos(\lambda - \lambda_0) \sin \beta \\
k &= 20.496''
\end{align*}
\]

• over a year, star traces ellipse with axes $k$, $k \sin \beta$
Diurnal Stellar Aberration

- due to Earth’s axial rotation

\[
\Delta \alpha = -k \cos \varphi \cosh \sec \delta \\
\Delta \delta = -k \cos \varphi \sinh \sin \delta \\
k = -0.3198''
\]

\( \varphi \) – latitude, \( h \) – hour angle

- both annual and diurnal aberration shift entire field
  - needed for pointing a telescope with \(~1''\) accuracy
  - not of concern if required relative positional accuracy is \(>0.05''\)
Parallactic Motion

- **annual**
  - due to Earth’s annual orbital motion around Sun
  - i.e., same as in measuring trigonometric parallax

- **diurnal**
  - difference in zenith distance seen from Earth’s surface vs. from geocenter
  - amplitude: \( \frac{R_{\text{Earth}}}{d} \cos l \)
    - \( d \) – distance to object in units of \( R_{\text{Earth}} \)
    - \( l \) – latitude
    - 57’ for Moon, 8.8” for Sun
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Apparent Proper Motion
(from Lecture 3)

- $\mu \equiv$ annual proper motion
- $\theta \equiv$ position angle (PA) of proper motion

Barnard’s star, 1.8 pc, $\mu = 10.3''/yr$
Apparent Proper Motion
(from Lecture 3)

- $\mu \equiv$ annual proper motion
- $\theta \equiv$ position angle (PA) of proper motion

\[ \mu_\delta = \mu \cos \theta \]
\[ \mu_\alpha \cos \delta = \mu \sin \theta \]

- relative to Local Standard of Rest (LSR) or heliocenter
  - average reference frame from motions of nearby stars
  - Sun’s peculiar velocity w.r.t. LSR is 19.5 km/s in direction of 18.0h, 30°
Radial Motion (or Velocity)

- blueshift, redshift
- modulated by
  - Sun’s motion ($v = 220 \text{ km/s}$) around center of Galaxy
  - Earth’s motion around Sun ($v = 29.8 \text{ km/s}$)
  - Earth’s axial rotation ($v = 0.47\cos\lambda \text{ km/s}$)

$$v_r = c \frac{\Delta \lambda}{\lambda} \approx cz$$
Stellar Space Motions

- $UVW$ galactic coordinates
- at Sun’s location, most stars in disk have
  - $V \sim -20$ km/s
  - $U \sim 0$ km/s
  - $W \sim 0$ km/s
UVW Motions Example: Young Moving Groups

Zuckerman & Song (2004)
Young Moving Groups

- <0.1 Gyr-old stars cluster in a distinct $UVW$ "box"

*Zuckerman & Song (2004)*
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Tangent-Plane Geometry

- sky is curved; images of it are flat
  - measured $X$, $Y$ positions do not translate linearly to angular coordinates on sky
  - see map of Moving Groups (slide 17)

RA, DEC to $X$, $Y$

$$
X = -\frac{1}{S} \frac{\cos \delta \sin(\alpha - \alpha_t)}{\sin \delta \sin \delta_t + \cos \delta \cos \delta_t \cos(\alpha - \alpha_t)}
$$

$$
Y = \frac{1}{S} \frac{\sin \delta \cos \delta_t - \cos \delta \sin \delta_t \cos(\alpha - \alpha_t)}{\sin \delta \sin \delta_t + \cos \delta \cos \delta_t \cos(\alpha - \alpha_t)}
$$

$\alpha_t$, $\delta_t$ – RA, DEC of tangent point (where image is tangent to sky; usually, center of image)

$X$, $Y$ to RA, DEC

$$
\theta = \tan(-X/Y) ; \quad \phi = \tan\left(S\sqrt{X^2 + Y^2}\right)
$$

$$
\alpha = \alpha_t + \arcsin\left(\frac{\sin \phi \sin \theta}{\cos \delta \cos \phi - \sin \delta \sin \phi \cos \theta}\right)
$$

$$
\delta = \arcsin(\sin \delta \cos \phi + \cos \delta \sin \phi \cos \theta)
$$
UVW Motions Example: Young Moving Groups

Zuckerman & Song (2004)
Astrometry: Limitations
(from Lecture 8)

• limiting precision
  – $\delta r \sim \text{FWHM} / \text{SNR}$
  – unattainable in practice

• systematic effects
  – differential atmospheric refraction
  – pixel sampling
  – focal plane curvature, distortion
Atmospheric Refraction
(from Lecture 4)

\[ n (3200 \text{ Å}) = 1.0003049 \]
\[ n (5400 \text{ Å}) = 1.0002929 \]
\[ n (10,000 \text{ Å}) = 1.0002890 \]

Differential atmospheric refraction \( D \) between 3200 Å and 5400 Å
Astrometry: Pixel Sampling
(from Lecture 8)

- \( r = \text{FWHM} / (\text{pixel size}) \)
- \( r < 1.5 \): under-sampled
- Nyquist sampling: \( r \sim 2 \) (\( r = 2.355 \), precisely)
  - optimal SNR, error rejection, positional precision
- \( r > 2 \) desirable for best photometry, astrometry on bright point sources
Fig. 1. Six PSFs compared. The left two panels show bright and faint stars imaged with the LONEOS camera if it had 3 μm pixels. The middle panels show the same two stars as they may appear in actual LONEOS data; i.e., poorly sampled. Note, however, how the fainter star is helped by the large pixel size. The right two panels again show the same two LONEOS stars but they are now not centered within a pixel but near a corner. The vertical scales are not common to all panels. The high-S/N, well-sampled star (top left) looks like and can be well approximated by a Gaussian PSF. Even a well-sampled, lower S/N image can be analyzed using the same technique with some success. The poorly sampled images (4 panels on the right hand side) cannot be well approximated by a Gaussian function.
Focal Plane Distortion: Beam Tilt

Figure 4.10: A diagram of the intersection between the detector image plane \((x', y')\) and the tilted beam plane \((x_b, y_b)\). The two planes intersect along the line AB at an angle \(\zeta\). The orientation of AB with respect to the detector \(x\) axis in the image plane is \(\phi\). The projections of the \(x_b\) and \(y_b\) unit vectors onto the image plane are denoted as \(x_t\) and \(y_t\). Right angles are marked with square corners.

- in reality focal plane has a high-order curvature

Metchev (2006)
Distortion of the Wide Field Camera on the HST Advanced Camera for Surveys
Distortion of the Wide Field Camera on the HST Advanced Camera for Surveys

WFC1 PIXEL AREA MAP
Normalized to 0.05 arcsecond square pixel

HST ACS Instrument Handbook 28
- HST focal plane
Example: HR 8799 Planets

Marois et al. (2008)
HR 8799bcd: Detection through Angular Differential Imaging

Angular Differential Imaging (ADI)

Keck Ks-band 10s integration
ADI-processed 10s integration
Combined ADI

Total integration time (s) = 10

Marois et al. (2008)
HR 8799bcd: Astrometry

Marois et al. (2008)
HR 8799bcd: Orbital Motions

Metchev et al. (2009); Konopacky et al. (2010)