

## Stellar Explosions

Three assumptions make modeling tractable analytically. First, the pressure of the expanding remnant is dominated by radiation pressure. Second, the energy radiated from the surface and by gamma emission from radioactivity in the interior are small compared to the total energy. Third, spherical symmetry is valid to zeroth order.

The first law of thermodynamics  $dE + PdV = TdS = dQ$  is

$$\dot{E} + P\dot{V} = -\frac{\partial L}{\partial M} + \dot{\epsilon} \quad (1)$$

where  $E = aT^4V$  and  $P = aT^4/3$  are the energy per gram and the pressure. Dots represent time derivatives. The volume per gram is  $V = 1/\rho$ ,  $\dot{\epsilon}$  is the energy input per gram per second from radioactivity,  $L(r, t)$  is the luminosity, and  $M(r, t)$  is the mass. Define  $x = r(M, t)/R(t)$  as a dimensionless Lagrangian radius containing mass  $M$  at time  $t$  and radius  $r$ .  $R(t)$  is the surface (defined below). The density at a given enclosed mass scales in time with  $R(t)^{-3}$  since

$$M(x) = 4\pi R(t)^3 \int_0^x \rho(r, t) x^2 dx \quad (2)$$

is independent of time. We define a dimensionless density  $\eta(x)$  by

$$\rho(r, t) \equiv 1/V = \rho_o \eta(x) \left( \frac{R_o}{R(t)} \right)^3, \quad (3)$$

where  $\rho_o \equiv \rho(0, 0)$  and  $R_o \equiv R(0)$ . Thus  $\dot{V}/V = 3\dot{R}/R$ .

If the right-hand side of Eq. (1) was neglected, adiabaticity applies and  $\dot{T}/T = -\dot{R}/R$ . This is the major time dependence of  $T$ . Now define

$$T^4(r, t) = \Psi(x) \phi(t) T_o^4 R_o^4 / R^4(t) \quad (4)$$

so  $\dot{T}/T = -\dot{R}/R + \dot{\phi}/4\phi$ . Here,  $T_o \equiv T(0, 0)$ . The luminosity becomes

$$L(r, t) = -\frac{4\pi r^2}{3} \frac{ac}{\kappa\rho} \frac{\partial T^4}{\partial r} = -\frac{4\pi x^2}{3} \frac{ac}{\kappa\rho_o\eta} \phi T_o^4 R_o \frac{d\Psi}{dx}, \quad (5)$$

where  $\kappa$  is the opacity. The equation of energy, Eq. (1) becomes

$$-\frac{3\dot{\epsilon}\rho_o^2 R_o^2}{acT_o^4} \left( \frac{\eta}{\phi\Psi} \right) + \frac{3R_o^3\rho_o}{c} \frac{\dot{\phi}}{\phi R} = \frac{1}{\Psi x^2} \frac{d}{dx} \left( \frac{x^2}{\eta\kappa} \frac{d\Psi}{dx} \right). \quad (6)$$

For the moment, ignore the radioactive contributions. The opacity is dominated by electron scattering, for which  $\kappa=\text{constant}$ , except very near the surface where the temperature is low. There, a Kramer's opacity ;with  $\kappa \propto \rho; T^{-3.5} \propto \eta(\Psi\phi)^{-7/8} R^{1/2}$  would be appropriate. For simplicity, we take  $\kappa = \kappa_o$ . The differential equation (6) is now separable. It becomes

$$\frac{3\rho_o\kappa_o R_o^3}{c} \frac{\dot{\phi}}{\phi R} = -\alpha = \frac{1}{\Psi x^2} \frac{d}{dx} \left( \frac{x^2}{\eta} \frac{d\Psi}{dx} \right). \quad (7)$$

The time dependence is easily solved for:

$$\phi(t) = \exp \left[ -\frac{\alpha c}{6\rho_o\kappa_o R_o^3} \int_0^t R(t) dt \right] = \exp \left[ -\frac{1}{2R_o\tau_o} \int_0^t R(t) dt \right] \quad (8)$$

where the usual diffusion timescale is

$$\tau_o = \frac{3R_o^2\rho_o\kappa_o}{\alpha c}. \quad (9)$$

The explicit time behavior depends upon  $R(t)$  (see below).

## Spatial Solutions

We next examine the spatial solution, which permits us to evaluate the eigenvalue  $\alpha$ . The boundary conditions on  $\Psi$  are easily given. First, at the origin,  $\Psi(0) = 1$  and  $\Psi'(0) = 0$ . At the surface we use the Eddington boundary condition

$$\left( \frac{T}{T_e} \right)^4 = \frac{\Psi}{\Psi_e} = \frac{3}{4} \left( \tau + \frac{2}{3} \right) \quad (10)$$

where  $\tau(r) = -\int_r^\infty \kappa\rho dr$  is the optical depth and the subscript  $e$  refers to the effective radiating surface where  $\tau = 2/3$  and  $r = R$  or  $x = 1$ . From Eq. (10) we have that

$$\Psi(1) \equiv \Psi(\tau = 0) = \frac{1}{2}\Psi_e \quad (11)$$

and by differentiation, using  $\Psi' = d\Psi/dx$ ,

$$\Psi'(1) = -\frac{3}{4}\Psi_e (\kappa\rho)_{x=1} R. \quad (12)$$

Combining these two, and setting  $\kappa = \kappa_o$ , we have

$$\Psi(1) = -\frac{2}{3} \frac{\Psi'}{\kappa_o \rho R} \Big|_{x=1}. \quad (13)$$

With  $\eta\kappa$  constant, the solution of Eq. (7) is a polytrope of index 1:

$$\Psi(x) = \frac{\sin \sqrt{\alpha} x}{\sqrt{\alpha} x}. \quad (14)$$

The boundary condition Eq. (13) implies

$$\frac{\sin \sqrt{\alpha}}{\sqrt{\alpha}} = -\frac{2}{3} \frac{1}{\kappa_o \rho R} \left( \cos \sqrt{\alpha} - \frac{\sin \sqrt{\alpha}}{\sqrt{\alpha}} \right), \quad (15)$$

or

$$\sqrt{\alpha} \simeq \pi \left( 1 - \frac{2}{3} \frac{1}{\kappa_o \rho R} \right). \quad (16)$$

Note that  $\kappa_o \rho R$  is the total optical depth in the case of uniform density. For  $\kappa_o \rho R \rightarrow \infty$ , we have  $\Psi(1) = 0$  and  $\alpha = \pi^2$ . Only if the total optical depth is less than about 10 is there significant deviation from this result, and this generally occurs only after several months. The leading correction is  $\Psi(1) \simeq 2/(3\kappa_o \rho R)$ . In what follows, we will simply impose the outer boundary condition as  $\Psi(1) = 0$ .

Suppose the density  $\eta$  is not constant. For  $\eta = \Psi^m$  the solution is related to the polytropic Lane-Emden solution  $\theta_n$  for the index  $n$ :

$$\Psi = \theta_n^n, \quad \eta = \theta_n^{n-1}, \quad x = \xi \sqrt{n/\alpha}, \quad n = 1/(1-m). \quad (17)$$

With the boundary condition  $\Psi(1) = 0$ , some cases are shown in Table 1. Note that  $m = 1/3(2/3)[4/5]$  is the  $n = 3/2(3)[5]$  polytrope, and the eigenvalue  $\alpha$  is  $\alpha = n\xi_1^2$ . The case  $m = 4/5$  has  $\alpha = \infty$ ; a density spike in the center with a zero density mantle – not physically very relevant.

The polytropic solutions all have  $\eta$  (density) decreasing with distance from the center. A shell-like behavior can also be modelled:

$$\eta = \frac{1}{1 - \beta x^2}; \quad \Psi = 1 + \gamma x^2 + \delta x^4. \quad (18)$$

The boundary condition on the outside becomes  $\gamma + \delta = -1$ . Physical solutions are possible for two cases:

- a)  $\alpha = (140/9)(1 - \sqrt{8/35}) \simeq 8.119, \beta = \alpha/28, \gamma = -\alpha/6, \delta = -1 + \alpha/6$ .
- b)  $\alpha = 6, \beta = 3/5, \gamma = -1, \delta = 0$ .

The ratios of densities of the surface and center are  $\eta(1)/\eta(0) = 1/(1 - \beta) = 2.5$  (1.41).

**Table 1:** Spatial solutions to Eq. (7)

$\eta$	$\alpha$	$I_M$	$I_K$	$I_T$	$\alpha I_M$	$I_M/I_K$
$\eta = \theta_3^2$	142.7	0.0132	0.0024	0.00615	1.882	5.42
$\eta = \theta_{3/2}^{1/2}$	20.03	0.151	0.071	0.0556	3.028	2.15
$\eta = \eta_o$	$\pi^2$	1/3	1/5	0.101	3.290	5/3
a: $\eta = (1 - \beta x^2)^{-1}$	8.119	0.409	0.260	0.113	3.303	1.572
b: $\eta = (1 - \beta x^2)^{-1}$	6	0.553	0.366	0.133	3.319	1.510

### Temporal Evolution

The temporal evolution will yield the light curves. When the shock emerges at the surface  $R_o$  at time  $t = 0$ , the energy is nearly evenly divided between thermal and kinetic energies:

$$E_T(t) = \int_0^R aT^4 4\pi r^2 dr = 4\pi R_o^3 aT_o^4 \frac{R_o}{R} \phi(t) I_T = E_T(0) \phi(t) \frac{R_o}{R}, \quad (19)$$

$$E_K(t) = \frac{1}{2} \int_0^R \rho v^2 4\pi r^2 dr = 2\pi \rho_o R_o^3 \dot{R}^2 I_K = E_K(0) \frac{\dot{R}_o^2}{\dot{R}^2}, \quad (20)$$

where  $I_T = \int_0^1 \Psi x^2 dx$ ,  $I_K = \int_0^1 \eta x^4 dx$ ,  $v = dr/dt = x\dot{R}$  and  $\dot{R}_o$  is its initial value. The total energy  $E_{SN}$  is a constant of the motion if the energy lost in radiation or gained from radioactivity is negligible, and is given by its initial value:

$$E_{SN} = E_T(0) + E_K(0) \simeq 2E_T(0). \quad (21)$$

Reference to Eq. (5) allows us to now write the luminosity as

$$L(t) = -\frac{4\pi}{3} \frac{ac}{\kappa_o \rho_o} R_o T_o^4 \left( \frac{\Psi'}{\eta} \right)_{x=1} \phi(t) = -\frac{4\pi c I_M R_o E_T(0)}{3\kappa_o M I_T} \left( \frac{\Psi'}{\eta} \right)_{x=1}. \quad (22)$$

The total ejected mass is  $M = \int_0^R 4\pi \rho r^2 dr = 4\pi \rho_o R_o^3 I_M$ , where  $I_M = \int_0^1 \eta x^2 dx$ . From Eq. (7) we have the identity

$$-\alpha I_T = \left( \frac{x^2 \Psi'}{\eta} \right)_{x=1} = \left( \frac{\Psi'}{\eta} \right)_{x=1}, \quad (23)$$

so, with Eq. (22), we find

$$L(t) = \frac{2\pi c E_{SN}}{3\kappa_o M} \alpha I_M R_o \phi(t) = \frac{E_{SN} \phi(t)}{2\tau_o}. \quad (24)$$

Conservation of energy  $E_{SN} = E_T(t) + E_K(t)$  yields

$$\dot{R}^2 = \dot{R}_o^2 (2 - \phi R_o/R). \quad (25)$$

At  $t = 0$ ,  $\phi(t)R_o/R(t) \simeq 1$ ,  $R$  increases linearly with time:  $R \simeq R_o + \dot{R}_o t$ . The expansion timescale initially is

$$\tau_h = \frac{R_o}{\dot{R}_o} = 2.5 \times 10^5 \left( \frac{R_o}{10^{14} \text{ cm}} \right) \left( \frac{4 \times 10^8 \text{ cm s}^{-1}}{\dot{R}_o} \right) \text{ s}. \quad (26)$$

However, after a time of several  $\tau_h$ , *i.e.*, at most days, the term  $\phi R_o/R \ll 1$ . The expansion is still linear in time, but  $\sqrt{2}$  times faster than it was initially: the thermal energy has been converted into kinetic energy of expansion. A reasonable approximation is

$$R(t) = R_o + \sqrt{2} \dot{R}_o t. \quad (27)$$

We are finally able to solve for the function  $\phi(t)$ : using Eq. (8) we have

$$\phi(t) = \exp \left[ -\frac{t}{\tau_o} - \frac{t^2}{\sqrt{2} \tau_o \tau_h} \right]. \quad (28)$$

Since  $\tau_o \gg \tau_h$ ,  $\phi$  initially decreases exponentially, but before much decay has occurred, the decrease steepens into a Gaussian with time constant

$$\tau_{decay} = \sqrt{\sqrt{2} \tau_o \tau_h}. \quad (29)$$

It is useful to now express things in terms of the quantities  $M$ ,  $R_o$  and  $E_{SN}$  which we can hope to extract from a supernova's light curve. For example, the diffusion time becomes, using Eqs. (9),

$$\tau_o = 1.6 \times 10^7 \frac{\kappa_o \pi^2}{\alpha I_M \text{ cm}^2 \text{ g}^{-1}} \left( \frac{M}{M_\odot} \right) \left( \frac{10^{14} \text{ cm}}{R_o} \right) \text{ s}. \quad (30)$$

The maximum observed velocities in the ejecta's spectra are, using Eqs. (20) and (21),

$$v_{max} \simeq \sqrt{2} \dot{R}_o = \sqrt{\frac{2E_{SN} I_M}{M I_K}} \simeq 10^9 \sqrt{\frac{E_{SN}}{10^{51} \text{ ergs}} \frac{M_\odot}{M} \frac{I_M}{I_K}} \text{ cm s}^{-1}. \quad (31)$$

Thus, the decay time, given by Eq. (29), becomes

$$\tau_{decay} = 1.7 \times 10^6 \sqrt{\frac{\kappa_o \pi^2}{\alpha I_M \text{ cm}^2 \text{ g}^{-1}}} \left(\frac{M}{M_\odot}\right)^{3/4} \left(\frac{10^{51} \text{ ergs}}{E_{SN}}\right)^{1/4} \left(\frac{I_K}{I_M}\right)^{1/4} \text{ s.} \quad (32)$$

Additionally, we may write the luminosity, Eq. (24), as

$$L = 3.1 \times 10^{43} \frac{\alpha I_M \text{ cm}^2 \text{ g}^{-1}}{\kappa_o \pi^2} \frac{E_{SN}}{10^{51} \text{ ergs}} \frac{M_\odot}{M} \frac{R_o}{10^{14} \text{ cm}} \phi \text{ erg s}^{-1}, \quad (33)$$

and the effective temperature,  $T_e = (L/4\pi\sigma R^2)^{1/4}$ , as

$$T_e = 4.6 \times 10^4 \left(\frac{\alpha I_M \text{ cm}^2 \text{ g}^{-1}}{\kappa_o \pi^2}\right)^{1/4} \left(\frac{E_{SN} M_\odot}{M 10^{51} \text{ ergs}}\right)^{1/4} \left(\frac{10^{14} \text{ cm}}{R_o}\right)^{1/4} \left(\frac{R_o}{R}\right)^{1/2} \phi^{1/4} \text{ K} \quad (34)$$

The largest time dependence in the effective temperature is due to the expansion. The density and opacity dependences in the ejecta have a relatively small effect on observables, through  $\sqrt{\alpha I_M}$  and  $(I_M/I_K)^{1/4}$ , which are insensitive to assumptions about  $\eta$  (Table I). For the last two cases shown,  $I_M = \beta^{-3/2}(\tanh^{-1}(\sqrt{\beta}) - \sqrt{\beta})$  and  $I_K = (I_M - 1/3)/\beta$ . For a Population I composition,  $\kappa_o \simeq 0.33 \text{ cm}^2 \text{ g}^{-1}$ , and the combination  $\kappa_o \pi^2 / \alpha I_M \simeq 1$ . Similarly,  $I_M/I_K \simeq 1.7$ .

Summarizing: measurement of the largest velocities in the spectrum gives the ratio  $E_{SN}/M$  from Eq. (31). The effective temperature gives the initial radius, by Eq. (34). The peak luminosity, through Eq. (33) then can give the distance to the supernova. Finally, measurement of the decay timescale provide an estimate of the ejected mass, through Eq. (32).

## Radioactive Heating

Typical Type 2 light curves do not show the effects of radioactive energy input until a year or more after peak light, but in Type 1s, it dominates the light curve from the first few days. Much more radioactive nickel is produced, and the ejecta mass and initial radius are much smaller.

Choose a simple parametrization of the radioactivity:

$$\dot{\epsilon} = \dot{\epsilon}_o \xi(x) e^{-t/\tau_r} \quad (35)$$

where  $\epsilon_o$  is the energy released per gram of radioactive nuclei per second,  $\xi$  is the density distribution of the radioactive nuclei, and  $\tau_r$  is the radioactive

decay timescale. For  $^{56}\text{Ni} \rightarrow ^{56}\text{Co}$ ,  $\epsilon_o = 4.78 \times 10^{10} \text{ erg g}^{-1} \text{ s}^{-1}$  and  $\tau_r = 7.6 \times 10^5 \text{ s}$ . For  $^{56}\text{Co} \rightarrow ^{56}\text{Fe}$ ,  $\epsilon_o = 7.97 \times 10^9 \text{ erg g}^{-1} \text{ s}^{-1}$ , and  $\tau_r = 9.82 \times 10^6 \text{ s}$ . In Type 1 events, however, the exploding envelope is transparent to much of the cobalt decay gamma rays and positrons, making a simple characterization of  $\dot{\epsilon}$  difficult. This might have contributed to a mistaken identification with the decay time of Cf in some supernovae. For Ni-Co-Fe decay,

$$\dot{\epsilon} = \xi(x) \left[ \dot{\epsilon}_o^{Ni} e^{-t/\tau_r^{Ni}} + \dot{\epsilon}_o^{Co} \left( 1 - e^{-t/\tau_r^{Ni}} \right) e^{-t/\tau_r^{Co}} \right]. \quad (36)$$

The energy input from Co exceeds that from Ni when

$$\frac{t}{\tau_{Ni}} \simeq \frac{\ln \left( \dot{\epsilon}_o^{Ni} / \dot{\epsilon}_o^{Co} \right) + e^{-t/\tau_{Ni}}}{1 - \tau_{Ni} / \tau_{Co}} \simeq 2.08 \quad (37)$$

or about 18 days. After  $4\tau_{Ni}$ , the Ni decay is only producing 1/8 of the total energy, and Eq. (35) becomes a good approximation. The decay ( $^{56}\text{Ni} \rightarrow ^{56}\text{Fe}$ ) liberates 0.12 MeV/nucleon or about  $2 \times 10^{50}$  ergs per solar mass of radioactive material, which does not destroy the homologous expansion, since it is such a small fraction of  $E_{SN}$ .

Using

$$\dot{\epsilon} = \dot{\epsilon}_o \xi(x) f(t), \quad (38)$$

one notes that if the function  $\dot{\epsilon}\eta/\phi\Psi$  is approximately independent of  $x$ , we can separate Eq. (6) as before. Taking  $\xi(x)\eta(x)/\Psi(x) \equiv b$ , a constant, is tantamount to concentrating radioactive nuclei in the center, not an obviously bad assumption. The initial mass of radioactive nuclei becomes

$$M_r = 4\pi R_o^3 \rho_o \int_0^1 \xi(x) \eta(x) x^2 dx = b \frac{I_T}{I_M} M. \quad (39)$$

We may now rewrite Eq. (6) as

$$\dot{\phi} + \phi \frac{R(t)}{\tau_o R_o} = f(t) \frac{R(t)}{R_o} \frac{2\dot{\epsilon}_o M_r}{E_{SN}}. \quad (40)$$

Define

$$\dot{u} = R(t) / (R_o \tau_o), \quad u = t/\tau_o + t^2/\tau_{decay}^2.$$

The second follows from  $R(t)$  (Eq. (27)). The solution of Eq. (40) is

$$\phi = \bar{\epsilon} e^{-u} \int_0^u e^u f du + e^{-u} \quad (41)$$

where

$$\bar{\epsilon} = \frac{2\dot{\epsilon}_o M_r \tau_o}{E_{SN}} = \frac{M_r \dot{\epsilon}_o}{L(0)} = 3.1 \frac{M_r M}{M_\odot^2} \frac{10^{14} \text{ cm}}{R_o} \frac{10^{51} \text{ ergs}}{E_{SN}}, \quad (42)$$

evaluated for nickel decay. This quantity measures the importance of radioactivity to the light curve, especially at early times. For times  $t < \tau_{Ni}$ , we have  $f \propto e^{-t/\tau_{Ni}}$ , and for  $t > 3\tau_{Ni}$ , we have  $f \propto e^{-t/\tau_{Co}}$ . For extremely early times,  $u \ll 1$ , we may use the fact that  $\tau_{Ni}, \tau_{decay} \ll \tau_o$  to find

$$\phi = 1 + (\bar{\epsilon} - 1)u \quad u \ll 1. \quad (43)$$

For Type 1 supernovae, the initial radius is very small. For the envelope of the presupernova to be in hydrostatic equilibrium, the presupernova luminosity must be less than the Eddington limit  $4\pi cGM_{core}/\kappa$ . This implies that the envelope radius is limited to

$$R_o < \sqrt{\frac{cGM_{core}}{\sigma\kappa}} \frac{1}{T_e^2} = 2.1 \times 10^{13} \left( \frac{M_{core}}{1.4M_\odot} \right)^{1/2} \left( \frac{6000\text{K}}{T_e} \right)^2 \text{ cm}. \quad (44)$$

Eq. (42) thus shows that  $\bar{\epsilon} \gg 1$ , and we expect the bolometric luminosity to initially increase after the shock reaches the photosphere. For Type 2 supernovae with a red supergiant progenitor (we note the important exception of SN1987A!),  $R_o \sim 10^{14}$  cm,  $M_r < 0.1M_\odot$ , and so  $\bar{\epsilon} \leq 1$  and the luminosity will begin to fall almost immediately.

Over very long times, Eq. (41) becomes

$$\phi \simeq e^{-u} + \bar{\epsilon}(f - e^{-u}). \quad u \gg 1 \quad (45)$$

Even for  $\bar{\epsilon} < 1$  the radioactive term is important. For  $t \gg \tau_{decay}$ ,  $u \propto t^2$  and  $f \propto \exp(-t/\tau_{Co})$ . The  $f\bar{\epsilon}$  term will eventually dominate.

The luminosity function Eq. (41) has the property that it reaches a maximum when  $\phi = f\bar{\epsilon}$ , *i.e.*, the light curve peaks when it has the same value as the radioactive term, at least when  $\bar{\epsilon} > 1$ . If  $\bar{\epsilon} < 1$ , the maximum is at  $t = 0$ . The width of the maximum in the light curve, which may of course be measured, can be estimated from

$$\Delta_{peak} \simeq \left( \frac{d^2\phi}{dt^2} \right)_{t_{max}}^{-1/2} = \tau_{decay} \sqrt{\frac{\tau_{Co}}{2\phi_{max}t_{max}}} \quad (46)$$

where  $t_{max}$  is the time when the luminosity is maximum. Thus, the larger the decay time, the broader the peak of the light curve.



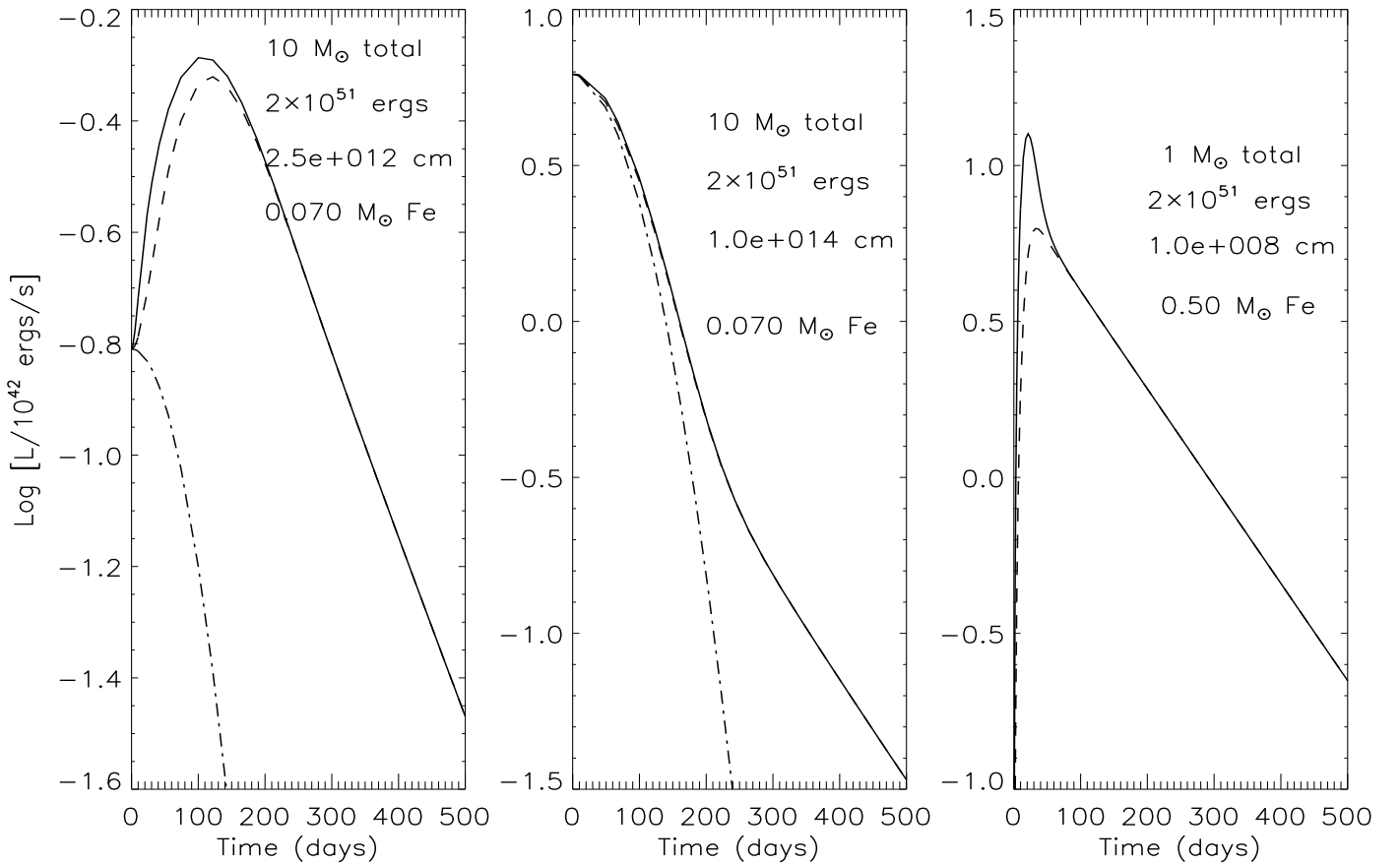
In summary, the luminosity, when radioactive input is considered, is

$$\begin{aligned}
 L &= 3.1 \times 10^{43} \frac{E_{SN}}{10^{51} \text{ ergs}} \frac{M_{\odot}}{M} \frac{R_o}{10^{14} \text{ cm}} \left[ 1 + \bar{\epsilon} \int_0^u e^u f du \right] e^{-u} \text{ ergs} \\
 &= 3.1 \times 10^{43} \left[ \left( \frac{3.1}{.5} \right) \frac{M_r}{M_{\odot}} \int_0^u e^u f du + \frac{E_{SN}}{10^{51} \text{ ergs}} \frac{M_{\odot}}{M} \frac{R_o}{10^{14} \text{ cm}} \right] e^{-u} \text{ ergs}.
 \end{aligned} \tag{47}$$

The upper (lower) number in the second equation corresponds to Ni (Co) decay. We have used representative values for the opacity and the spatial part of the solution. The more massive the ejecta, and the more compact the envelope, the more important radioactivity is. For consistency, the maximum amount of energy input by radioactivity ( $\sum \dot{\epsilon}_o^r \tau_r$ ) should be small compared to  $E_{SN}$ . In the case of the Type 2 supernovae, with massive and extended envelopes, radioactivity becomes important at late enough times. From Eq. (45) this occurs when  $e^{-u} \approx \bar{\epsilon} f$  or

$$\left( \frac{t}{\tau_{decay}} \right)^2 - \frac{t}{\tau_{Co}} \simeq \ln \left[ 1 + 40 \frac{E_{SN}}{10^{51} \text{ erg}} \frac{M_{\odot}}{M} \frac{0.1 M_{\odot}}{M_r} \frac{R_o}{10^{14} \text{ cm}} \right], \tag{48}$$

which is generally  $t \simeq (1 - 2)\tau_{decay} \simeq 100 - 200$  days.



The figure shows three typical cases of supernova light curves. The left panel is for the Type 2 supernova SN 1987A. A more typical 'red giant' Type 2 supernova is in the middle panel. The right-hand panel is for a Type 1 supernova. The solid lines are the full solutions, the dashed lines assume the radioactivity is due only to Co decay, and the dash-dot lines assume no radioactive energy input.

### Application to SN 1987A

SN 1987A appeared to be subluminous for a Type 2. Eq. (33) immediately suggests that the presupernova envelope must have been smaller than usual; in fact, from photographic identification, the progenitor was a blue, not a red, supergiant. Neutrinos being observed less than 4 hours before the optical display is corroborative evidence: the time necessary for the shock wave with a velocity equal to the largest velocities initially observed in the spectrum, about  $5 \times 10^8 \text{ cm s}^{-1}$ , to traverse the envelope is equal to two hours (it takes 2 hours after shock breakout for the optical emission to occur) if  $R_o \simeq 3 \times 10^{12} \text{ cm}$ . The bolometric luminosity remained nearly constant for the first few weeks and then increased. The shape of the initial light curve thus cannot completely determine the envelope properties.

The initial luminosity, based on the assumed distance to the Large Magellanic Cloud of 50 kpc, was about  $1.8 \times 10^{41} \text{ ergs s}^{-1}$ . Coupled with Eq. (47), evaluated at  $t = 0$ ,  $R_o \simeq 2.5 \times 10^{12} \text{ cm}$  gives an energy to mass ratio of  $E_{SN}/M \simeq .18 \times 10^{51} \text{ ergs}/M_\odot$ . Eq. (30) says that the diffusion time was about  $\tau_o \simeq 20.2(M/M_\odot) \text{ yrs}$ , and the decay time  $\tau_{decay} \simeq 26.6\sqrt{M/M_\odot}$  days, for nominal values of the opacity,  $\alpha$ ,  $I_M$  and  $I_K$ .

After several months, the light curve became exponential decay with a halflife equal to that of Co decay. The total mass of radioactive nuclei may be found from the absolute position of the light curve's tail:

$$L(t \rightarrow \infty) \simeq 1.55 \times 10^{43} \left( \frac{M_r}{M_\odot} \right) e^{-t/\tau_{Co}}, \quad (49)$$

which gives  $M_r \simeq 0.075 M_\odot$ . Therefore, for cobalt decay, one finds

$$\bar{\epsilon} = \frac{2\dot{\epsilon}_{Co} M_r \tau_o}{E_{SN}} = \frac{\dot{\epsilon}_{Co} M_r}{L(0)} \simeq 8.5, \quad (50)$$

and for nickel,  $\bar{\epsilon} \simeq 40$ . However, Eq. (43) demonstrates that until  $\bar{\epsilon}u \simeq 1$ , or  $t \simeq \tau_{decay}/\sqrt{\bar{\epsilon}}$ , the radioactive heating will have little effect. In the case of SN1987A, this occurs (for nickel)  $32\sqrt{M/41M_\odot}$  days after the explosion.

The fact that the light curve began to increase due to radioactive heating after about 1 1/2 weeks is already an indication that the ejecta mass is in the range of 5–10  $M_{\odot}$ .

The intermediate times are more complicated to describe analytically, but if we have simple radioactive decay of one species, so that  $f = \exp(-t/\tau_{Co})$ , one can describe the peak conditions according to:

$$u_{peak} + y \approx 1.8 - 2.0; \quad \phi_{peak} \approx 0.17\bar{\epsilon}/y. \quad (51)$$

This approximation is valid when  $\bar{\epsilon} \gg 1$  and  $0.3 < y < 1.5$  with

$$y = \frac{\tau_{decay}}{2\tau_{Co}} = 0.15 \left( \frac{M}{M_{\odot}} \right)^{3/4} \left( \frac{10^{51}\text{ergs}}{E_{SN}} \right)^{1/4}. \quad (52)$$

Thus,  $t/\tau_{Co} \simeq 2y\sqrt{u}$ . From the observation that the light curve peaked around  $t_{peak} = 90$  days after the explosion, and the relations

$$u_{peak} = (t_{peak}/\tau_{decay})^2 \simeq 12.3 (M_{\odot}/M); \quad y = 0.23\sqrt{M/M_{\odot}},$$

we deduce that  $M \approx 9 - 11M_{\odot}$ . We used the energy to mass ratio described above. The mass scales as the square root of the energy to mass ratio, and is inversely proportional to the opacity, whose value was taken to be equal to that for a solar composition gas ( $=0.33$ ). For metal-rich ejecta the opacity is greater than for a solar gas, making our estimate of the mass too high.

It is worth noting that the light curve for SN 1987A, although agreeing with the general behavior of the solutions we have discussed, differs from them in some important respects. First, the observed peak value of the luminosity was about 5 times the initial luminosity (excluding transient effects due to the ultraviolet burst) compared to about  $\phi_{peak} \simeq 0.17\bar{\epsilon}/y \simeq 2$  times in the analytic solution. Second, the peak of the observed light curve does not lie exactly on the asymptotic radioactive decay line, and it lacks the symmetry and breadth of the analytic solutions. This behavior implies the existence of additional energy sources or uneven distribution of radioactive components. One additional energy source is due to recombination in to the cooling wave that eats inward into the ejecta. This contribution cannot be easily included in the analytic solution.

## Application to Type 1 Supernovae

In the case of Type 1 supernovae, involving the explosion of white dwarfs, the radius is of order 1000 km. The explosion energy is again of order  $2 \times 10^{51}$  ergs. Using Eq. (42), we find for Ni that  $\bar{\epsilon} \approx 4 \times 10^5$  if the ejecta mass is  $1 M_\odot$  and  $1/2$  is Ni. We also note that the parameter

$$y = \tau_{decay}/2\tau_{Ni} = 1.3 \left( \frac{M}{M_\odot} \right)^{3/4} \left( \frac{10^{51} \text{ erg s}^{-1}}{E_{SN}} \right)^{1/4} \approx 1.1 \quad (53)$$

for Ni decay. Eq. (51) then suggests that the peak in the light curve is reached when  $u_{peak} \approx 0.6$  and  $\phi_{peak} \approx 0.16\bar{\epsilon}$ . The maximum luminosity is therefore about  $0.16M_r\dot{\epsilon}_{Ni} = 8 \times 10^{42}$  ergs/s, which is completely independent of  $R$  and is also insensitive to the explosion energy and the ejected mass. This is what makes Type 1 SN so valuable as distance indicators. Note that the light curve peak is noticeably shifted from the explosion time by a few weeks:

$$t_{peak} = \sqrt{u_{peak}\tau_{decay}} \approx 17 \text{ days.}$$