Homework # 2 Solutions

There is an error in the assignment, in that the case $n = 0$ is an exception. The Lane-Emden equation is

$$\theta'' = -2\theta'/\xi - \theta^n.$$

Assume a solution near the origin of the form

$$\theta = 1 + \alpha\theta + \beta\theta^2 + \dots$$

We have

$$\theta' = \alpha + 2\beta\xi, \qquad \theta'' = 2\beta$$

keeping lowest-order terms. Substituting:

$$2\beta = -2\alpha/\xi - 4\beta - 1$$

keeping all terms to order ξ^0. Clearly, $\alpha = 0$ and $\beta = -1/6$.

Near the point where θ vanishes, we have

$$\theta'' = -2\theta'/\xi$$

which must be positive since θ' is negative. This means θ is concave upwards at this point. An exception is $n = 0$, the last term is 1, and the sum of the two terms is negative.

If a step $d\xi = -\theta_0/\theta_0'$ is taken at a point where $\theta = \theta_0$ is positive, a Taylor expansion of θ gives

$$\theta = \theta_0 + \theta_0' d\xi + \frac{1}{2}\theta_0'' (d\xi)^2 + \dots = \theta_0 - \theta_0 + \frac{1}{2}\theta_0'' (d\xi)^2$$

which is positive since θ_0'' is positive. So taking this step is guaranteed to keep the new value of θ positive.