

PHY 521 – Stars

Exam #1 – 24 March 2008

In all cases, partial credit will be assigned. Some physical constants are provided at the end of the exam. There are 100 points total, check the reverse of this page. You are allowed to use the notes, any text, and the internet to find relevant information. Give your answers completely: don't say this is given by eq. xx of the notes, for example.

1. (15 points) Estimate the central pressure and temperature of main sequence stars 10 times more massive and ten times less massive than the Sun. You may assume these stars have solar composition. List your assumptions and show your work.

2. (25 points) Draw a temperature-density plot to explore the equation of state of an ideal electron gas. Assume that there is 1 electron for every two baryons in matter. Label the axes with cgs values (i.e., g/cm^3 and K) for the baryon density and the temperature. You should make this a log-log graph.
 - 2.1. Sketch approximate divisions between a) the extremely relativistic (ER) and non-relativistic (NR) regimes and b) the extremely degenerate (ED) and non-degenerate (ND) regimes.
 - 2.2. Quantitatively explain the location of these boundaries with one or two equations for each.
 - 2.3. Now sketch a curve in the figure to separate regions in which pairs of electrons and positrons are and are not important to the total pressure, assuming thermal equilibrium. Also quantitatively explain the equation of this line (with equations).
 - 2.4. Show where the centers of Betelgeuse, the Sun, Sirius B, Jupiter and the Earth are located in this figure. Also show where the visible surface of the Sun would be located.

3. (10 points) Sketch the mass-radius relation for white dwarfs. Assume an electron fraction of $Y_e = 1/2$. You don't have to solve this exactly, you may use approximations for different limits. Show any equations you use to draw the curve.

4. (20 points) Explain the origin of the nuclear virial theorem, which describes the most bound nucleus in nature, for a given neutron/proton ratio: $E_{surf} = 2E_{Coul}$.

Use the simple mass formula described in class. Estimate the binding energy of the most bound nucleus when $Z/A=1/2$ and show your work.

In nuclear statistical equilibrium, if you approximated the distribution of nuclei by a Gaussian,

$$n(A) = n(A_B) \exp \left[- (A - A_B)^2 / \Delta^2 \right],$$

where A_B is the mass number of the most-bound nucleus and Δ is the width of the distribution, what are A_B and Δ if you assume $Z = A/2$ and use the simple mass formula described in class? Also assume the nuclear partition function $G(A, Z) = 1$ for all nuclei.

5. (15 points) Why do nuclear reactions in stars generally involve fusion of the lightest available nuclei into heavier nuclei? Given the general validity of this statement, it seems contradictory that massive stars on the main sequence burn H to He utilizing the CNO cycle rather than the p-p cycle. Explain quantitatively why this occurs.
6. (5 points) In the advanced stages of evolution of massive stars, i.e., C burning and beyond, how can one estimate the temperature and duration of each nuclear burning stage? (A qualitative answer is fine, indicating the physical principles involved, so no numerical calculations are needed.)
7. (10 points) In the case of an isothermal perfect gas (assume it to be both non-relativistic and non-degenerate), how does the mass depend upon the radius? What physical situation might this case be applicable to?

Some physical constants: $m_e c^2 = 0.511$ MeV, $m_b c^2 = 939$ MeV,
 $k_B = 1.38 \cdot 10^{-16}$ erg/K, 1 MeV = $1.6 \cdot 10^{-6}$ erg, $\hbar c = 197.3$ MeV-fm,
 1 fm = 10^{-13} cm, $G = 6.67 \cdot 10^{-8}$ erg cm g⁻², $c = 3 \cdot 10^{10}$ cm s⁻¹,
 $M_\odot = 2 \cdot 10^{33}$ g, $R_\odot = 7 \cdot 10^{10}$ cm, $e^2 = 1.44$ MeV-fm.