PHY 521 – Stars

Exam #2 – due 17 May 2010

1. The equation of radiative transfer for coherent isotropic scattering for a plane-parallel gray atmosphere is

$$\mu \frac{dI\left(\mu, \tau\right)}{d\tau} = I\left(\mu, \tau\right) - J\left(\tau\right).$$

Assume a solution of the form

$$I\left(\mu, \tau\right) = a\left(b + \tau + \mu + \frac{c}{1 + d\mu}e^{-f\tau}\right),$$

where (a, b, c, d, f) are constants. Find the mean intensity $J(\tau)$, the flux $F(\tau)$, and the pressure integral $K(\tau)$. Show that the condition of radiative equilibrium in the atmosphere, $F(\tau) = F =$ constant, also implies that $K(\tau) = J(\tau)/3$, $a = 3F/4$ and d is infinitesimally small but finite.

The exact solution of the gray atmosphere problem can be written

$$J\left(\tau\right) = \frac{3}{4}F\left[\tau + q\left(\tau\right)\right],$$

where $q(\tau)$ is the Hopf function. The exact solution has $q(0) = 1/\sqrt{3}$ and $q(\infty) = 0.710446$. Find the constants (b, c, d, f) that result in radiative equilibrium and the correct Hopf function in the limits of $\tau = 0$ and $\tau = \infty$.

For this model, how much brighter would the center of the stellar disk appear compared to the limb of a star?

Can this model generally satisfy the condition that there is no incoming radiation at the stellar surface?

2. The problem concerns the cooling of white dwarfs. Suppose that the conductivity in a white dwarf interior is so large, because of electron degeneracy, that the star is nearly isothermal with temperature T. However, there is an atmosphereand so the visible surface of the star has an effective temperature $T_{eff} \neq T$. Approximate the effect of the atmosphere through

$$T_{eff} = KT^{7/8},$$

where K is a constant. Assume the mass M and radius R of the white dwarf remain constant as it cools. Although the pressure in the white dwarf interior is dominated by degenerate electrons, the heat capacity in the interior is dominated by ions. Approximate the total heat capacity by

$$C_V = \frac{3}{2}Nk_B$$

where N is the number of ions in the interior. Also assume that the ions are all C^{12}.

The white dwarf cools primarily by thermal emission of radiation from the surface. What is the slope $d\ln L/d\ln T_{eff}$ of the cooling trajectory in the Hertzsprung-Russel diagram? How does this compare to the slope for lower Main Sequence stars?

Suppose that white dwarfs have been born at a constant rate over the age of the Galaxy T_G. What is the smallest luminosity and effective temperature a white dwarf with mass of 1 $M_\odot$ and radius 1000 km would have at the present time? Finally, sketch the behavior of the relative number of white dwarfs in our Galaxy in a logarithmic luminosity interval as a function of luminosity.

3. Consider a binary star with masses M_1^0 and M_2 initially in a circular orbit with semi-major axis a_0. A supernova occurs in star 1 resulting in an immediate mass loss of $\Delta M = M_1^0 - M_1$. If the supernova remnant receives no kick, i.e., its spatial velocity immediately after the explosion is the same as before, what is the maximum mass loss that can occur yet still result in a bound binary? Express your answer in terms of a fraction of the total initial mass $M = M_1^0 + M_2$.

In some neutron star binaries, it is obvious that explosions have occurred in which more mass loss occurred than predicted above, yet the binaries are still bound? How could this happen?

4. In this problem consider an explosion that occurs within a star of uniform density ρ, uniform temperature T, and mass M. Immediately after the shock wave from the explosion reaches the surface, the stellar gas is so hot that it is radiation-pressure dominated. Energy diffuses out of the star by radiative transport, and the opacity is due mostly to electron scattering. The total energy imparted to the matter is of order $E_{SN} = \varepsilon_r V$, where ε_r is the energy density of radiation and V is the initial stellar volume $4\pi R_0^3/3$ with R_0 the initial stellar radius. Assuming the stellar structure radiative transfer equation holds throughout the star, show that the luminosity at the stellar surface is

$$L \propto \frac{E_{SN}}{M} \frac{R_0}{\kappa}$$

where κ is the opacity.

Estimate the ratio E_{SN}/M for a Type Ia supernova.

Assuming that this ratio is about the same for a gravitational collapse (Type II) supernova, why is the initial luminosity of a Type Ia supernovae much, much fainter than a Type II?

At late times, both types of supernovae exhibit light curves that exponentially decay with a time constant of about 80 days. Why is this?

5. Assume the core of a red giant star is isothermal, with $T \simeq 10^7$ K and a core mass $M_{core} \simeq 1$ $M_\odot$. Assume the pressure in the core is due to an ideal gas. Beyond its boundary a thin layer produces essentially all the star's luminosity. Also assume the red giant's envelope is radiative and use the equations of hydrostatic equilibrium, the ideal gas law, and radiative transfer to show that the stellar radius is approximately 1000 $R_\odot$.