Supernovae and Neutrinos

The photonic and kinetic energies of a Type 2 supernovae are only a fraction of the total energy released. The optical energy is about 10^{43} erg/s for a month, or 10^{49} erg. The kinetic energy is about 10 $M_\odot$ at velocities 0.01 c^2, or 10^{51} erg. But the total energy released is the binding energy of the neutron star that's formed: $3GM^2/5R$ or $3 \cdot 10^{53}$ ergs. This energy emerges mostly in the form of neutrinos.

Neutrino Trapping

The discovery of neutral currents in the early 1970's led to the modern picture in which stellar collapse produces to a lepton-rich (electrons and neutrinos) remnant plus an ejected mantle powered by a shock and heat from neutrinos leaking out of the core. Electron capture, which otherwise would change the matter from $Y_e = Z/A \simeq 0.42$ before collapse to $Y_e \leq 0.1$ in cold, catalyzed neutron star matter, is suppressed because neutrino trapping occurs shortly after the collapse begins. We use $g_A = 1.253$,

$$\sigma_o = (4/\pi)\,(m_e c/\hbar)^4 \left(G_F/m_e c^2\right)^2 = 1.76 \times 10^{-44}\ \text{cm}^2.$$

1. Neutral current scattering by free nucleons,

$$\nu + n \overset{Z}{\rightarrow} \nu + n, \qquad \nu + p \overset{Z}{\rightarrow} \nu + p, \tag{1}$$

$$\sigma_n = \frac{\sigma_o}{4}\left(\frac{E_\nu}{m_e c^2}\right)^2 = 1.7 \times 10^{-44} E_\nu^2\ \text{cm}^2 \qquad ND,$$

$$\sigma_n = \frac{\pi^2 \sigma_o}{64}\left(1 + 2g_A^2\right)\left(\frac{T}{m_e c^2}\right)^2\left(\frac{E_\nu}{p_F c}\right)\left(\frac{m_n c^2}{\epsilon_F}\right) = 2.1\times 10^{-45} T^2 E_\nu \frac{\rho_s}{\rho}\ \text{cm}^2 \quad D.$$

2. Neutral current coherent scattering by heavy nuclei,

$$\nu + (Z, A) \overset{Z}{\rightarrow} \nu + (Z, A)\,, \tag{2}$$

$$\sigma_A = \frac{\sigma_o}{16}\left(\frac{E_\nu}{m_e c^2}\right)^2 \left[A + Z\left(4\sin^2\theta_W - 2\right)\right]^2 \simeq 4.2 \times 10^{-45} N^2 E_\nu^2\ \text{cm}^{-2}.$$

3. Charged current nucleon absorption,

$$\nu_e + n \overset{W}{\rightarrow} p + e^- \tag{3}$$

$$\sigma_a = \frac{\sigma_o}{2}\left(1+3g_A^2\right) Y_e \left(\frac{E_\nu}{m_e c^2}\right)^2 = 1.9\times 10^{-43} Y_e E_\nu^2 \ \text{cm}^2 \qquad ND,$$

$$\sigma_a = \frac{3\pi^2\sigma_o}{128}\left(1+3g_A^2\right)\left(\frac{T}{m_e c^2}\right)^2\left(\frac{m_n c^2}{\epsilon_F}\right)\left(\frac{Y_e}{1-Y_e}\right)^{1/3} =$$

$$1.43\times 10^{-42}\left(\frac{Y_e}{1-Y_e}\right)^{1/3} T^2 \left(\frac{\rho_s}{\rho}\right)^{2/3} \ \text{cm}^2. \qquad D$$

4. Charged and neutral current electron-neutrino scattering,

$$\nu + e^- \overset{W,Z}{\to} \nu + e^- . \tag{4}$$

$$\sigma_e = 0.1\sigma_o \left(\frac{E_\nu}{m_e c^2}\right)^2 \frac{E_\nu}{\mu_e}. \tag{5}$$

The largest opacity is due to coherent scattering, and the neutrino mean free path ($\lambda_\nu = 1/<\sigma\rho>$) is

$$\lambda_\nu \simeq \frac{60}{\rho_{12}}\left(6X_n + 5X_p + A\left(1-x_N\right)^2 X_A\right)^{-1}\left(\frac{10\ \text{MeV}}{E_\nu}\right)^2 \text{km}. \tag{6}$$

We used a simple form for σ_α and $X_\alpha = 1 - X_n - X_p - X_H$. We find $\lambda_\nu = R \simeq (3M/4\pi\rho)^{1/3}$ when $\rho \simeq 3\times 10^{10}$ g cm^{-3}. The actual trapping density occurs when the ν diffusion time is smaller than the collapse timescale:

$$\tau_d = \frac{3R^2}{\pi^2 c\lambda_\nu} \simeq .03\rho_{12} \ \text{s} = \tau_c = \sqrt{33}\left(\frac{3}{8\pi G\rho}\right)^{1/2} \simeq 7.5\times 10^{-3}\rho_{12}^{-1/2} \ \text{s}.$$

The factor $\sqrt{33}$ follows from self similar models. Thus $\rho \simeq 4\times 10^{11}$ g cm^{-3}.

$Y_L = Y_e + Y_\nu$ is frozen at values near 0.4, and the collapse is essentially adiabatic (no loss of neutrinos, no change in electron fraction). The adiabat has a rather low entropy, $s \simeq 1$ per nucleon, because of extensive neutrino cooling during the post-carbon burning stages of the precollapse iron core. At the beginning of collapse, $T \approx 0.7$ MeV, $\rho \approx 4\times 10^9$ g cm^{-3}, and $Y_e \simeq 0.42$. The translational entropy of nuclei per *Fe nucleus* is

$$S_{nuc} = \frac{5}{2} + \log\left[\left(\frac{56mT}{2\pi\hbar^2}\right)^{3/2}\frac{1}{n_{56}}\right] \simeq 17.$$

Excited nuclear states will contribute an entropy per *nucleus* of

$$S_{ex} = 56\left(\frac{\pi^2}{2}\right)\left(\frac{T}{T_F}\right) \simeq 4.8,$$

where $T_F \simeq 35$ MeV is the fermi energy of nuclear matter. The entropy of electrons per *electron* is

$$S_e = \pi^2 \frac{T}{\mu_e} \simeq 1.1.$$

There is a dilute vapor of neutrons also, with an entropy per *nucleon* of

$$S_{vapor} = \frac{5}{2} + \log\left[\left(\frac{mT}{2\pi\hbar^2}\right)^{3/2} \frac{2}{n_n}\right] \simeq 12.9.$$

Summing these contributions, we find per baryon

$$s = X_H \frac{S_{nuc} + S_{ex}}{56} + S_e Y_e + S_{vapor} X_n \simeq 0.92.$$

(The solar center has an entropy per baryon in nuclei alone of 16.5).

Consider a *one-zone* collapse model using $R \simeq (3M/4\pi\rho)^{1/3}$ to understand the evolution of Y_e and Y_L. Take

$$\frac{\partial \ln \rho}{\partial t} = \frac{1}{\tau_c}. \tag{7}$$

The first law of thermodynamics is

$$\dot{q} = T\dot{s} + \sum_i \mu_i \dot{Y}_i =< E_{\nu,esc} > \left(\dot{Y}_e + \dot{Y}_\nu\right), \tag{8}$$

where i refers to nucleons, nuclei and leptons. The heat change $\dot{q}$ is due to the escape of neutrinos with average energy $< E_{\nu,esc} >$. In nuclear statistical equilibrium, the sum is ($\hat{\mu} = \mu_n - \mu_p$)

$$\sum_i \mu_i \dot{Y}_i = \dot{Y}_e\left(\mu_e - \hat{\mu}\right) + \dot{Y}_\nu \mu_\nu. \tag{9}$$

$$T\dot{s} = -\dot{Y}_e\left(\mu_e - \hat{\mu} - \mu_\nu\right) - \left(\dot{Y}_e + \dot{Y}_\nu\right)\left(\mu_\nu - < E_{\nu,esc} >\right).$$

The first term is the entropy generation from being out of beta equilibrium while the second is due to losing νs. If νs freely escape, $\mu_\nu = 0$, and then

$$T\dot{s} = -\dot{Y}_e\left(\mu_e - \hat{\mu} - < E_{\nu,esc} >\right) \qquad \text{free escape.}$$

When νs are fully trapped, $< E_{\nu,esc} >= \mu_\nu$ because only νs at the top of the Fermi sea will escape. μ_ν increases until beta equilibrium is established and reverse reactions balance forward ones: entropy generation is halted.

For changes in Y_e, it is sufficient to consider only ν capture on free protons, which in the case of degenerate e^-s and freely escaping νs, is

$$\dot{Y}_e = -\frac{3}{5}\left(\frac{\mu_e}{m_e c^2}\right)^2 \mathcal{R} n Y_e X_p \sigma_o c = 488\rho_{12} Y_e X_p \mu_e^2 \mathcal{R}\ \mathrm{s}^{-1}, \qquad (10)$$

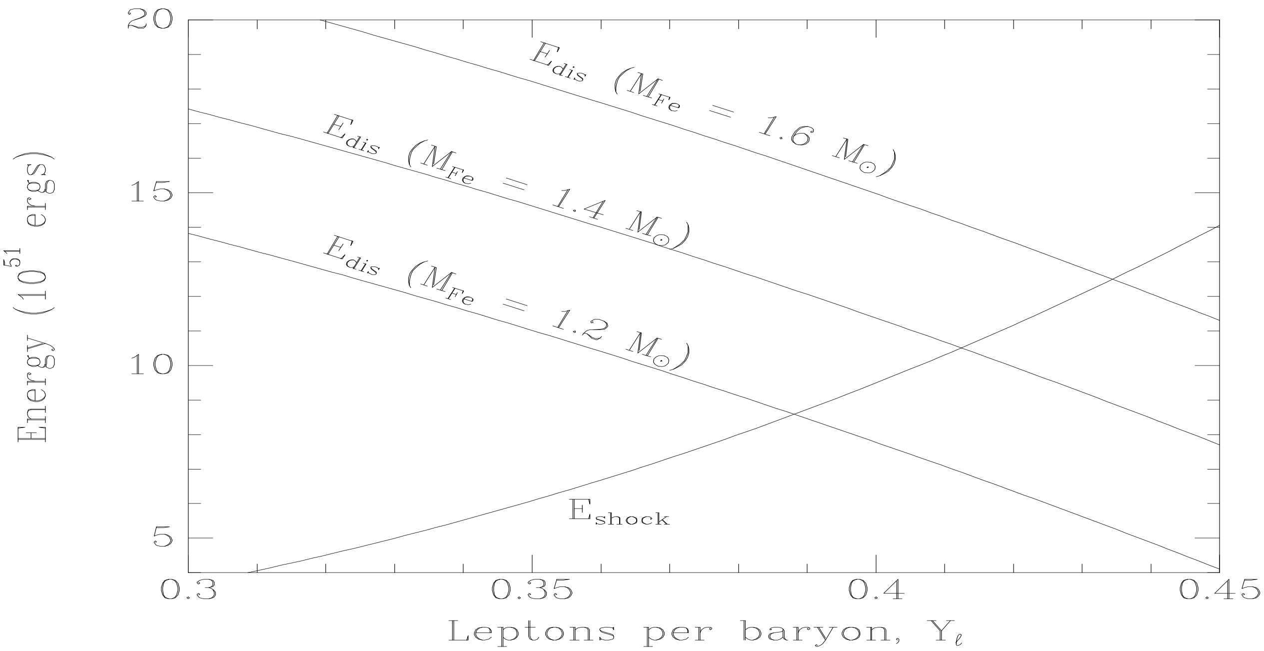
where $\mathcal{R} = 1 - e^{(\mu_e - \hat{\mu} - \mu_\nu)/T}$ accounts for reverse processes that force the rates to 0 in $\beta-$equilibrium. Y_L changes due to neutrino loss. Early, νs freely stream out; later they leak out via diffusion. We can approximate

$$\dot{Y}_L \approx \frac{-Y_L}{\tau_\nu}; \qquad \tau_\nu = \frac{R}{c}\sqrt{1 + \left(\frac{3R}{\lambda_\nu}\right)^2}.$$

There is great sensitivity of Y_e on X_p and T. For non-interacting protons, $X_p \propto \exp(\mu_p/T)$. $\partial\mu_p/\partial Y_e \simeq 90$ MeV, when the sum of bulk, surface and Coulomb energies is considered. The entropy change is closely coupled to Y_e such that T will increase with ρ. Thus the system is highly self-regulating: as Y_e decreases, μ_p also decreases, which reduces X_p, turning off the electron captures. The net result is that full $\beta-$equilibrium is established by $\rho_{12} = 1$, and Y_L is marginally less than the initial Y_e.

Nuclei are not dissociated during the collapse due to the low entropy and high Y_e. When the central density approaches ρ_s, the nuclei merge into a nucleon fluid that is relatively incompressible. Only above this density is stability restored and the collapse halted. The pressure is dominated by relativistic electrons and neutrinos, with an effective adiabatic index constant at about 1.30. We've seen that a self-similar solution exists in this case. The collapsing core thus separates into an inner, homologous ($v \propto r$) part, and an outer, supersonically infalling part that is left behind. The mass of the inner core is somewhat larger than the equivalent Chandrasekhar mass $\propto Y_L^2$. When the collapse is halted, a shock wave is produced at the boundary between the inner and outer parts of the core, because outside of this point, sound travels more slowly than the matter is moving. Due to energy conservation, the initial energy of the shock is well approximated by the binding energy of the inner core $\propto GM^{5/3} \propto Y_L^{10/3}$.

For the shock to be successful, it must propagate through the outer core, and eject the envelope. The shock's energy is dissipated by dissociation of nuclei in its path, which takes about 9 MeV/nucleon, or 18×10^{51} ergs/$M_\odot$. The mass of the outer core is that of the initial iron core minus the mass of the inner core. The success of the shock depends strongly on Y_L and the initial iron core mass. If the shock stalls, neutrino heating of matter behind the shock may eventually resuscitate it. Otherwise, the shock is forced back by ram pressure of infalling material, and a black hole would form.



Can the shock succeed? The mass traversed and dissociated by the shock dissociate it, are both proportional to $\propto (Y_{Fe}^2 - Y_L^2)$, where $Y_{Fe} \simeq 0.41-0.43$ is the effective lepton fraction in the precollapse iron core, and Y_L is the trapped lepton fraction. On the other hand, the available energy scales as $Y_L^{10/3}$, showing the importance of Y_L (see the figure). It is essential that M_{Fe} be relatively small. Calculations show that $Y_L \leq 0.36-0.38$, so the shock alone appears to fail to eject the mantle and envelope of the star.

Neutrino Winds

Hundreds of milliseconds after bounce, $L_\nu = L_{\bar{\nu}} \approx 10^{52}$ ergs s^{-1}. This is smaller than the peak flux after bounce (10^{54} ergs/sec) and also the Eddington flux that would promptly lift off the outer envelope:

$$L_{edd} = \frac{4\pi c G M}{\kappa} \simeq 8.6\times10^{54}\frac{M}{M_\odot}\left(\frac{10\ \mathrm{MeV}}{E_\nu}\right)^2\ \mathrm{erg\ s^{-1}}.$$

Still, the flux heats the material behind the shock. The ν cross section is about $\kappa = \kappa_o E_\nu^2$, where $\kappa_o = 5.8\times10^{-20}$ cm^2 g^{-1} MeV^{-2}. The inverse processes of electron and positron capture cool the matter, with rates proportional to T^6. The fluxes are small and the mantle is transparent, so the net energy deposited is small. No more than about 0.1% of the total neutrino energy can be absorbed, but this may be sufficient.

The neutrino spectrum is thermal with T_ν and average energy

$$< E_\nu^2 >= \frac{F_5\,(0)}{F_3\,(0)} T_\nu^2 = \frac{310\pi^2}{147} T_\nu^2 = 20.8 T_\nu^2.$$

The net heating rate per gram, assuming $n_{e^-} = n_{e^+}$ and $F_\nu = F_{\bar{\nu}}$, is

$$\dot{q} = \frac{7}{16}\kappa_o ac T_\nu^6 \left[\frac{f}{4}\left(\frac{R_\nu}{R}\right)^2 - \left(\frac{T}{T_\nu}\right)^6 \right] \frac{F_5\,(0)}{F_3\,(0)}.$$

The factor of 7/16 is due to Fermi statistics; $a = \pi^2/[15(\hbar c)^3]$. Here T refers to an irradiated parcel at radius R, while the temperature and radius of the "neutrinosphere", from which the νs effectively emerge, are R_ν and T_ν. f is an anisotropy factor due to the spherical geometry that varies between 1, in the radial free streaming limit, to 4 in the opaque limit.

If T is small, $\dot{q} > 0$ and $\dot{T} > 0$. As T increases, $\dot{q} \to 0$ where the maximum temperature (*kinetic equilibrium*) occurs:

$$T_{max} = T_\nu \left(\frac{R_\nu}{2R}\right)^{1/3} \simeq 0.5 T_\nu / R_7^{1/3}, \tag{11}$$

if $f \simeq 1$, $R_\nu \simeq 30$ km and $R_7 = R/10^7$ cm. T_{max} is proportional to T_ν and depends inversely on radius. But $\dot{q} \propto T_\nu^6$. Hence, this mechanism thrives if T_ν is high. This requires careful transport and hydrodynamics.

In diffusion from a star with constant density, the neutrinosphere temperature T_ν decreases with time. But due to compressional heating as the mantle settles (the *negative specific heat effect*), and joule heating during transport, one finds that T_ν increases with time, until compression ceases. Suppose,initially, the stellar radius is R_1 with density and temperature profiles $\rho \propto r^{-n}$ and $T \propto r^{-n/3}$. The neutrinosphere is located at $R_{\nu 1} \sim R_1 - \ell_1$, where the mean free path of neutrinos is $\ell \propto (T^2\rho)^{-1} \propto r^{5n/3}$. Now compress the star by a factor α such that $R_1 = \alpha R_2$. The density and temperature at a given radius will scale to $\alpha^m \rho$ and $\alpha^{m/3} T$, respectively, where m might be about 3. The relation $R_{\nu 2} = R_2 - \ell_2$ now gives us the new radial position of the neutrinosphere and its temperature. Assuming that $\alpha - 1$ is small, we can see that

$$\begin{aligned} \frac{R_{\nu 2}}{R_{\nu 1}} &\simeq 1 + (\alpha - 1)\,\frac{5m - 3 - 3q}{3q + 5n}, \\ \frac{T_{\nu 2}}{T_{\nu 1}} &\simeq 1 + (\alpha - 1)\,\frac{n + q\,(n + m)}{3q + 5n}, \end{aligned} \tag{12}$$

where $q = R_{\nu 1}/\ell_1 \geq 1$. No matter what value q has, $T_{\nu 2} > T_{\nu 1}$, because $m > 0$ and $n > 0$. The question of whether or not the neutrinosphere moves in or out in space is more problematical, but is irrelevant.

Even if there is a net cooling near the neutrinosphere, since $\kappa \propto T^2$, a decrease in T will move the neutrinosphere deeper, to *higher* T. Paradoxically, the hotter interior core is revealed due to the cooling.

Early on, about 20 ms after bounce, just after the bounce-shock might have failed, the matter accreted through the shock has a density near 10^{10} g cm^{-3} and is electron-rich. At these electron densities, electron capture loss is swift, with a characteristic capture time

$$\tau_{cap} \simeq 5\,(2\rho_{10}Y_e)^{-5/3} \ \text{ ms.} \tag{13}$$

The mantle loses pressure support and sinks, falling onto the proto-neutron star. However, self-similar arguments show the pre-shock matter is thinning out with time roughtly as

$$\rho_{pre} \propto \rho_{post} \propto r^{-3/2} t^{1-3\gamma/2} \propto r^{-3/2} t^{-1},$$

since $\gamma \simeq 4/3$. It will only be a matter of time before the accreted matter has a low density and cools much less quickly.

The rarefaction of the accreted matter means it eventually beomes radiation dominated, when

$$\rho \simeq \rho_{rd} = 4 \times 10^9 \left(\frac{T}{2.5 \text{ MeV}} \right)^3 \ \text{g cm}^{-3} \simeq 4 \times 10^9 \left(\frac{T_\nu}{5 \text{ MeV}} \right)^3 \frac{1}{R_7} \ \text{g cm}^{-3}.$$

For matter-pressure dominated material, the specific internal energy $\propto T \simeq T_{max} \simeq$ constant. The gravitational specific energy

$$-E_g = \frac{GM}{R} \simeq 2 \times 10^{19} \frac{M}{1.5 M_\odot} \frac{1}{R_7} \ \text{erg g}^{-1}$$

is independent of ρ, as is $E_{int} + E_g < 0$. However, the specific internal energy of radiation dominated matter increases with decreasing density. The density where $E_{int} = |E_g|$ is

$$\rho_{crit} = 7.4 \times 10^8 \left(\frac{T_\nu}{5 \text{ MeV}}\right)^4 \left(\frac{1}{R_7}\right)^{1/3} \text{ g cm}^{-3}. \tag{14}$$

When $\rho < \rho_{crit}$, the matter becomes unbound and explodes!

One must remember that this argument assumes that the heating to T_{max} is instantaneous, which is certainly unrealistic. A crude estimate of the heating time can be made by dividing $|E_g|$ by $\dot{q}$. We get

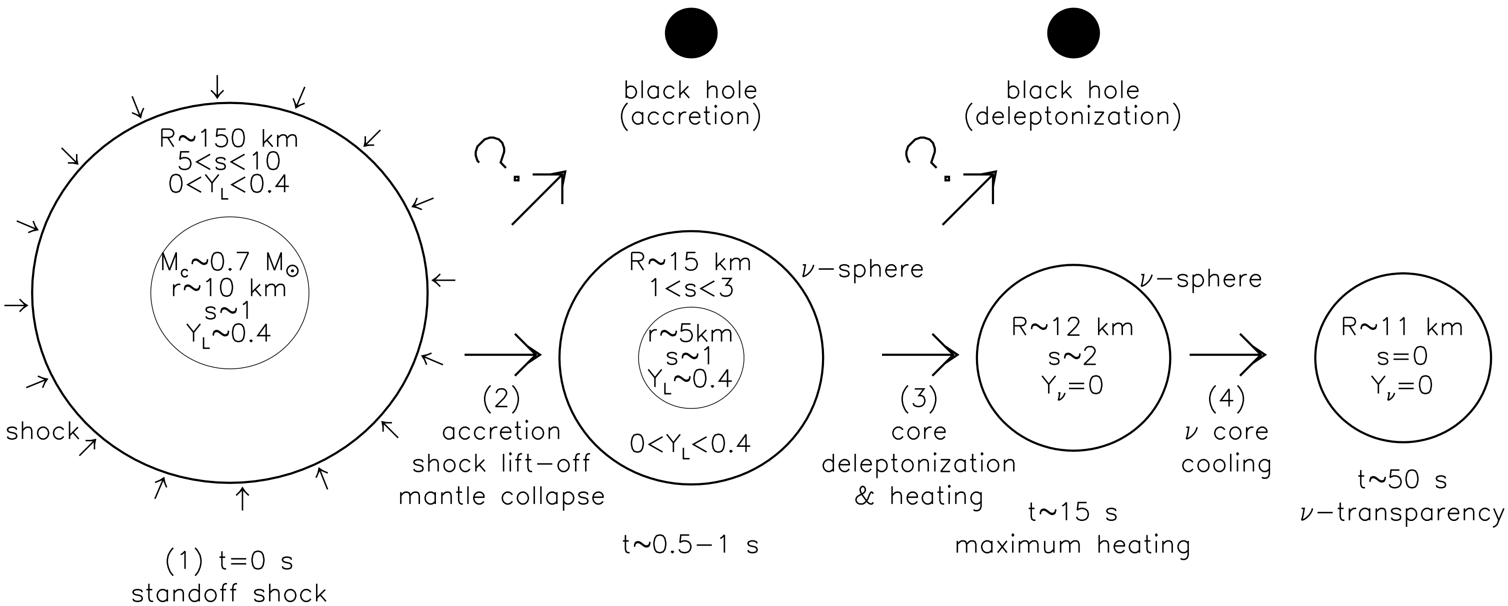
$$\tau_H \simeq \frac{10^{54} \text{ ergs s}^{-1}}{L_\nu} \frac{M}{1.5 M_\odot} \left(\frac{5 \text{ MeV}}{T_\nu}\right)^2 R_7 \text{ ms} \simeq 10 - 100 \text{ ms}, \tag{15}$$

where the neutrino luminosity is $L_\nu = 4\pi R_\nu^2 (7/64) ac T_\nu^4$. Thus, while the heating is certainly not instantaneous, it appears to be faster than some of the other relevant time scales, and fast enough to keep the post-shock matter near T_{max}.

Another point is that the binding energy of the mantle is constantly being buried through accretion in the proto-neutron star. Therefore, the binding energy that must be overcome to eject the envelope exterior to a given radius is constantly decreasing with time.

The time scales for T_ν increase, post-shock density decrease, binding energy decrease, capture turn-off, ram pressure increase, specific energy increase, etc., are all of the order of a few hundred milliseconds, while the heating is easily able to keep pace with all these changes. Time is on the supernova's side. If the neutrino fluxes are maintained and T_ν is reasonably high, all we have to do is wait a while for the shock to be revived by the core neutrinos.

The Birth of a Neutron Star



The accompanying figure shows the main stages of the life of a protoneutron star.

1. Following core bounce and shock passage, the star contains an unshocked, low entropy core of 0.5–0.7 $M_\odot$ with trapped neutrinos. This is surrounded by a low density, high entropy mantle accreting matter falling through the shock and rapidly losing energy due to electron captures and neutrino emission. The shock is momentarily stalled prior to an eventual explosion.

2. After about 0.5 s, accretion becomes much less important as the supernova becomes successful and the shock lifts off the stellar envelope. ν losses and deleptonization lead to pressure loss and mantle collapse. If enough accretion occurs, the star's mass could exceed M_{max} for hot, lepton-rich matter: if so a black hole forms and ν emission immediately halts.

3. This stage is dominated by neutrino diffusion causing deleptonization and heating of the core. $\nu-$nucleon absorption reactions set the diffusion timescale to 5–10 s. The maximum entropy reached in the core is about 2. As the core deleptonizes, strangeness, in the form of hyperons, a Bose condensate, or quarks, might apear. This could soften the equation of state and decrease M_{max} enough to form a black hole.

4. Following deleptonization is the long-term cooling phase. Although $\nu-$poor, thermally produced $\nu - \bar{\nu}$ pairs of all flavors are produced. The cooling timescale is determined by baryon and electron scattering of ν_μ and ν_τ, since ν_es remain more tightly coupled through absorption reactions. In approximately 50 s, as E_ν decreases, the star becomes essentially transparent to νs, and cools more rapidly.

5. (not shown) Following ν transparency, the core continues to cool by ν emission, but the star's crust cools less due to its low ν emissivity. The crust forms an insulating blanket preventing the star from coming to thermal equilibrium and keeps the surface relatively warm ($T \approx 3{\times}10^6$ K) for up to 100 years. This timescale is primarily sensitive to the neutron star's radius and the thermal conductivity of the mantle.

6. (not shown) Ultimately, the star achieves thermal equilibrium, a state of near isothermality, when the heat stored in the crust is depleted. The surface temperature T_{surf} is set by ν-rates in the star's core. If large (*rapid cooling*), T_{surf} becomes relatively small and the star is invisible. This could occur from direct Urca cooling from nucleons, hyperons, a Bose condensate or quarks. In *standard* cooling, relatively warm surfaces are maintained. Intermediate cases can occur if superfluids exist that suppress direct Urca cooling but don't eliminate it completely.

Analytic Models for Proto-Neutron Stars

We can examine some analytic models for proto-neutron star evolution. Let M be the gravitational mass, N the enclosed baryon mass, and F_ν and L_ν the number flux and luminosity of neutrinos. In GR, a term $e^\phi = \sqrt{-g_{00}}$ relates time at infinity τ with the coordinate time t. U is the internal energy per baryon.

$$\begin{aligned}
\frac{dP}{dr} &= -\frac{G\left(M+4\pi r^3 P\right)\left(\rho+P/c^2\right)}{r\left(r-2GM/c^2\right)} \\
\frac{dM}{dr} &= 4\pi r^2 \rho \\
\frac{dN}{dr} &= \frac{4\pi r^2 n}{\sqrt{1-2GM/rc^2}} \\
\frac{d\phi}{dP} &= -\frac{1}{P+\rho c^2} \\
\frac{dY_\nu}{d\tau} &= -e^{-\phi}\frac{\partial\left(4\pi r^2 F_\nu e^{\phi}\right)}{\partial N} + S_\nu \\
\frac{dY_e}{d\tau} &= -S_\nu \\
\frac{dU}{d\tau} &= -P\frac{d\left(1/n\right)}{d\tau} - e^{-2\phi}\frac{\partial L_\nu e^{2\phi}}{\partial N}
\end{aligned} \tag{16}$$

In the diffusion approximation, fluxes are driven by density gradients:

$$\begin{aligned}
F_\nu &= -\int_0^\infty \frac{c\lambda_\nu}{3}\frac{\partial n_\nu\left(E_\nu\right)}{\partial r} dE_\nu, \\
L_\nu &= -\int_0^\infty 4\pi r^2 \sum_i \frac{c\lambda_E^i}{3}\frac{\partial \epsilon_i\left(E_\nu\right)}{\partial r} dE_\nu.
\end{aligned} \tag{17}$$

The λ_ν and λ_E^i's are mean free paths for number and energy transport, respectively, and are functions of neutrino energy E_ν. $n_\nu(E_\nu)$ and $\epsilon_i(E_\nu)$ are the number and energy density of species $i = e, \mu$ at neutrino energy E_ν. GR corrections have been dropped for clarity.

We can combine Eq. (16) with the first law of thermodynamics to obtain the rate of change of the total lepton number and the entropy:

$$\begin{aligned}
n\frac{dY_L}{dt} = n\frac{dY_e}{dt} + \frac{dY_\nu}{dt} &= \frac{1}{r^2}\frac{\partial}{\partial r} r^2 F_\nu \\
nT\frac{ds}{dt} &= -\frac{1}{4\pi r^2}\frac{\partial L_\nu}{\partial r} - n \sum_{n,p,e,\nu} \mu_i \frac{dY_i}{dt}.
\end{aligned} \tag{18}$$

There are three main sources of opacity:

1. ν-nucleon absorption. Affects only $e-$types.

$$\lambda_{abs} \simeq 1.5 \left(\frac{4\rho_s}{\rho}\right)^{5/3} f_{FL} \left(\frac{E_{\nu o}}{E_\nu}\right)^2 s^{-2} \text{ cm}$$

   s is the entropy per baryon, $f_{FL} \simeq 2-3$ is a "Fermi-liquid" factor that corrects for interactions, and $E_{\nu o} \simeq 260$ MeV is typical at the beginning of deleptonization.
2. Neutrino-nucleon scattering. This elastic scattering affects all ν-types. $\lambda^e_{scn}/\lambda_{abs} \simeq 4(E_{\nu o}/E_\nu)$ for ν_e's. For ν_μ and ν_τ we have $\lambda^\mu_{scn} \simeq 2\lambda^e_{scn}$.
3. Neutrino-electron scattering. This inelastic scattering affects all types of neutrinos. The ratio $\lambda_{sce}/\lambda_{scn} \simeq 3s^2/f_{FL}$ in the degenerate limit and $19/f_{FL}$ when the neutrinos are nondegenerate.

Mean free paths for these processes are approximately:

1. $\lambda_{abs} \simeq 5$ cm, $\lambda_{abs} \propto E_\nu^{-2}$;
2. $\lambda_{scn} \simeq 20$ cm, $\lambda_{scn} \propto E_\nu^{-3}$;
3. $\lambda_{sce} \simeq 100$ cm, $\lambda_{sce} \propto E_\nu^{-3}$.

The opacities imply three kinds of fluxes:

1. a number flux F_ν of ν_es, dominated by absorption. With $\lambda_{abs} = \lambda_{abso}(E_{\nu o}/E_\nu)^2$, we have

$$F_\nu = \int_0^\infty \frac{c\lambda_{abs}}{3} \frac{\partial n_\nu (E_\nu)}{\partial r} dE_\nu = \frac{c\lambda_{abso} E^2_{\nu o}}{6\pi^2 (\hbar c)^3} \frac{\partial \mu_\nu}{\partial r} \equiv a \frac{\partial \mu_\nu}{\partial r}. \qquad (19)$$

2. an energy flux L^e_ν of ν_es, dominated by absorption:

$$L^e_\nu = \int_0^\infty 4\pi r^2 \frac{c\lambda_{abs}}{3} \frac{\partial \epsilon_\nu (E_\nu)}{\partial r} dE_\nu = 4\pi r^2 a \frac{\partial}{\partial r} \left(\frac{\pi^2 T^2}{6} + \mu^2_\nu\right). \qquad (20)$$

3. an energy flux L^μ_ν of ν_μs and ν_τs, dominated by scattering. Since $\lambda_{scn} >> \lambda_{sce}$, but only electron scattering is effective in transferring energy, we have $\lambda_{eff} = \sqrt{\lambda_{scn}\lambda_{sce}} = \lambda_{effo}(E_{\nu o}/E_\nu)^3$.

$$L^\mu_\nu = 16 \ln 2 \pi r^2 \frac{c\lambda_{effo} E^3_{\nu o}}{6\pi^2 (\hbar c)^3} \frac{\partial T}{\partial r} \equiv 4\pi r^2 b \frac{\partial T}{\partial r}, \qquad (21)$$

   where we used the fact that $\mu_\nu = 0$ for ν_μs and ν_τs. Note also that $b >> aT$ since $\lambda_{eff} >> \lambda_{abs}$.

Deleptonization Era

Deleptonization is dominated by number transport:

$$n\frac{dY_L}{dt} = n\frac{\partial Y_L}{\partial Y_\nu}\frac{dY_\nu}{dt} = \frac{a}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \mu_\nu}{\partial r}\right). \tag{22}$$

With the relations $nY_\nu \propto \mu_\nu^3$ and $\partial Y_L/\partial Y_\nu \propto \mu_\nu^{-1}$, valid for a degenerate neutrino gas, we establish

$$\mu_\nu\frac{\partial \mu_\nu}{\partial t} = \frac{a'}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \mu_\nu}{\partial r}\right). \tag{23}$$

We seek separable solutions of the form $\mu_\nu = E_{\nu o}\phi(t)\psi(r)$.

$$\frac{E_{\nu o}}{a'}\frac{\partial \phi}{\partial t} = \frac{1}{r^2\psi^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \psi}{\partial r}\right) = -\alpha, \tag{24}$$

where α is a separation constant. One sees that

$$\phi = 1 - t/\tau_d; \qquad \tau_d = \frac{3}{c\lambda_{abso}\alpha}\left(\frac{\partial Y_L}{\partial Y_\nu}\right)_o. \tag{25}$$

The factor $(\partial Y_L/Y_\nu)_o \simeq 3$ for $Y_{\nu o} \simeq 0.06$. The radial dependence is

$$-\alpha R^2\psi^2 = \frac{1}{x^2}\frac{\partial}{\partial x}\left(x^2\frac{\partial \psi}{\partial x}\right), \tag{26}$$

i.e., a Lane-Emden polytope of index 2. Thus $\alpha R^2 \simeq 19$ and

$$\tau_d \simeq \frac{R^2}{2c\lambda_{abso}} \simeq 7.5 < s^2 > \left(\frac{R}{15\ \text{km}}\right)^2\ \text{s} \simeq 15\ \text{s}. \tag{27}$$

The deleptonization is accompanied by heating in the core, up to a maximum of about 2 in entropy per baryon. Cooling is delayed.

$$nT\frac{ds}{dt} = a\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\left(\pi^2T^2/6\right)}{\partial r}\right) + \left(\frac{\partial \mu_\nu}{\partial r}\right)^2\right] + \frac{b}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right). \tag{28}$$

The first term is due to electrons, the second to the other neutrinos. There is heating or cooling depending on the sign of the T gradient, but the μ_ν gradients always lead to heating. When $\mu_\nu > T$, the first term dominates and we have net heating. When $\mu_\nu \simeq 0$, cooling dominates.

Thermal Cooling Era

The entropy is dominated by baryons for temperatures less than about 100 MeV. Thus, we may write

$$s \approx 2aT; \qquad a = \frac{1}{15}\frac{m^*}{m}\left(\frac{\rho_s}{\rho}\right)^{2/3} \ \mathrm{MeV}^{-1} = a_o\left(\frac{\rho_c}{\rho_s}\right)^{-2/3}, \tag{29}$$

where m^* is the effective nucleon mass and ρ_c is the central density. Thus

$$T_{max} = \frac{s_{max}}{2a_c} \simeq 37.8s\left(\frac{m}{2m^*}\right)\left(\frac{n_c}{4n_s}\right)^{2/3} \ \mathrm{MeV},$$

where a_o is the value of a at ρ_c at the beginning of cooling. Neglect the density dependence of m^* and use $m^* \simeq 0.5m$. Assuming separation of the time and radial dependence of the temperature $T = T_c\psi(r)\phi(t)$, and neglecting the first (electronic) term ($b >> aT$) in Eq. (28):

$$2anT_{max}\frac{d\phi}{dt} = \frac{b}{\psi r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) = -\alpha. \tag{30}$$

Thus the spatial solution is an $n = 1$ polytrope with $\alpha = \pi^2$, and

$$\phi = 1 - (t - \tau_d)/\tau_c, \tag{31}$$

where

$$\tau_c = \frac{ns_o}{4\ln 2}\frac{R^2}{\alpha}\frac{6\pi^2(\hbar c)^3}{c\lambda_{effo}E_{\nu o}^3} < s^3 > \simeq 7.5\left(\frac{R}{12 \ \mathrm{km}}\right)^2 < s^3 > \ \mathrm{s} \simeq 25 \ \mathrm{s}. \tag{32}$$

This result agrees with numerical calculations. Thus, in spite of the fact that the mean free paths that dominate cooling are much larger than those that dominate deleptonization, the higher initial entropy and more compact remnant force the cooling timescale to be longer than the deleptonization timescale. Note that the temperature during cooling decreases linearly with time, which also agrees with numerical calculations show.