

General Introduction

1. Luminosity, Flux and Magnitude

The luminosity L is an integral of the specific flux F_λ , the amount of energy at wavelength λ traversing a unit area per unit time:

$$L = 4\pi R^2 \int_0^\infty F_\lambda d\lambda.$$

Here R is the effective stellar radius. In the absence of any absorption between a star and the Earth, the incident energy flux is

$$f_\lambda = F_\lambda \left(\frac{R}{r}\right)^2,$$

where r is the distance to the star. In practice what is measured at the Earth's surface is modified by the transmissivity through the atmosphere A_λ and by the efficiency of the telescope E_λ whose collecting area is πa^2 :

$$b = \pi a^2 \int_0^\infty f_\lambda A_\lambda E_\lambda d\lambda.$$

b is the apparent brightness. The transmissivity is affected by the viewing angle.

Historically, astronomers measure brightness using magnitudes. Hipparchus denoted the brightest naked eye stars magnitude 1 and the dimmest magnitude 6. The response of the eye is logarithmic. By international agreement, the difference of 5 magnitudes is equivalent to a factor of 100:

$$\frac{b_1}{b_2} = 100^{(m_2 - m_1)/5} = 10^{4(m_2 - m_1)} = 2.512^{m_2 - m_1}.$$

m refers to a star's apparent magnitude.

A star's intrinsic brightness, or luminosity, is related to a star's apparent brightness through the inverse square law and a normalization. By convention, this relation is

$$\frac{B}{b} = \left(\frac{r}{10 \text{ pc}}\right)^2,$$

where B is the absolute brightness. Therefore,

$$M = m + 5 - 5 \log \frac{r}{\text{pc}}.$$

Since any detector has varying efficiency as a function of wavelength, one generally makes a bolometric correction to go from the recorded flux to the actual flux:

$$BC = 2.5 \log \frac{\text{incident energy flux}}{\text{recorded energy flux}}.$$

Then, the absolute bolometric magnitude M_b can be defined as

$$M_b = -2.5 \log \frac{L}{L_\odot} + 4.72,$$

where $L_\odot \simeq 3.9 \times 10^{33} \text{ erg s}^{-1}$.

2. Distances

The four most-used units of length in astronomy are the solar radius (R_\odot), astronomical unit (AU), light year, and parsec (pc):

- $R_\odot \simeq 7 \times 10^{10} \text{ cm}$
- $1 \text{ AU} = 1.5 \times 10^{13} \text{ cm}$
- $1 \text{ lt. yr.} = 9.3 \times 10^{17} \text{ cm}$
- $1 \text{ pc} = 3.1 \times 10^{18} \text{ cm} = 3.26 \text{ lt. yr.} = 1 \text{ AU}/\tan 1''$

The astronomical unit is based on the mean distance of the Earth from the Sun, a light year is the distance light travels in one year, and the parsec is the distance of a star whose apparent angular parallactic shift in three months is 1 second of arc.

3. Temperature

The **effective stellar temperature** is defined by

$$L = 4\pi R^2 \sigma T_{eff}^4.$$

Here, the radiation constant σ is defined by the integral

$$\sigma \equiv T^{-4} \int I_\lambda d\lambda = T^{-4} \int \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{ch/\lambda k_B T} - 1} d\lambda = \frac{2\pi^4 k_B^4}{15c^2 h^3} = 5.67 \times 10^{-5} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ deg}^{-4}.$$

The quantity $I_\lambda = \pi B_\lambda$ is the specific intensity of radiation at the wavelength λ , and B_λ is the Planck distribuion. We could equivalently define

$$I_\nu = \frac{2\pi h \nu^3}{c^2} \frac{1}{e^{h\nu/k_B T} - 1} = \pi B_\nu.$$

The energy density of radiation in thermodynamic equilibrium is

$$u = 4\pi \int B_\nu d\nu = 4\pi \int B_\lambda d\lambda = aT^4,$$

$$a = \frac{4\sigma}{c} = \frac{8\pi^5 k_B^4}{15c^3 h^3} = 7.565 \times 10^{-15} \text{ erg cm}^{-3} \text{ deg}^{-4}.$$

Thus, the effective temperature is obtained by matching the total power output of the star to a blackbody spectrum.

One can also try to match the shape of the spectra. One example is to look at the position of the peak of the spectrum. For a blackbody, this is

$$\lambda_{max} = \frac{hc}{5k_B T} = \frac{0.29 \text{ cm K}}{T}.$$

Matching this to the peak of the observed spectrum is one estimate of the **color temperature**. Usually, broadband spectral brightnesses or magnitudes can give good estimates of the stellar temperature. Frequently used broadband magnitudes are U , B and V , corresponding to the ultraviolet, blue and visual bands. The **color index** is the difference of two of these, $B - V$, and is often used instead of the stellar temperature to classify stars..

The **excitation temperature** is established from the observed relative populations of excited states in the stellar atmosphere. Since the gases in the atmosphere are Boltzmann gases (i.e., no interactions or degeneracy) in approximate thermal equilibrium, the relative population of two states i and j is given by the Saha equation

$$\frac{n_i}{n_j} = \frac{g_i}{g_j} e^{(\epsilon_i - \epsilon_j)/k_B T},$$

where g refers to the statistical weights of the states. For example, for a non-magnetized atomic state, $g_i = 2J_i + 1$ where J is the angular momentum. For hydrogen, $\epsilon_n = -13.6 \text{ eV}/n^2$ and $g_n = 2n^2$.

For states of ionization, the same principles apply, except that a third species, the electron, is involved. The **ionization temperature** is established with the Saha equation

$$\frac{n_{i+1} n_e}{n_i} = \frac{G_{i+1} g_e}{G_i} \frac{(2\pi m_e k_B T)^{3/2}}{h^3} e^{-\chi_i/k_B T}.$$

The ionization potential is χ_i . The statistical weights for each ionic state are sums over all the levels of that state:

$$G_i = g_{i,0} + g_{i,1} e^{-\epsilon_{i,1}/k_B T} + g_{i,2} e^{-\epsilon_{i,2}/k_B T} + \dots$$

4. Spectral Types

There is a correlation between conditions at the stellar surface and the spectral features observed. Originally, astronomers concocted a spectral typing based on the alphabet. However, the original 20-odd classes have been combined into the following scheme: O, B, A, F, G, K, M (*Oh, be a fine girl, kiss me* is a mnemonic dating from the mid-twentieth century to remember this.) These are in order from the hottest stars ($T \simeq 50,000$ K) to the coolest ones ($T \simeq 2200$ K). Some details of each type are:

- Class O ($T > 25,000$ K), ionized He dominates, other atoms with high ionization potential.
- Class B ($11,000$ K $< T < 25,000$ K, Balmer H and neutral He dominate, ionized C, O.
- Class A (7500 K $< T < 11,000$ K, H and ionized Mg dominate, but ionized Fe, Ti, Ca, &c become more important at lower temperatures.
- Class F (6000 K $< T < 7500$ K, ionized metals)
- Class G (5000 K $< T < 6000$ K, neutral metals, molecular CN, CH (Sun is G2)
- Class K (3500 K $< T < 5000$ K, molecular bands, neutral metals)
- Class M (2200 K $< T < 3500$ K, complex molecular oxide bands, TiO)

5. Physical Properties of Stars

Masses of stars can be determined in cases where the stars are in binaries. Kepler's Law

$$\frac{M_1 + M_2}{M_\odot} = \left(\frac{a}{\text{AU}}\right)^2 \left(\frac{\text{year}}{P}\right)^3$$

where M_1, M_2 are the stellar masses, and a and P are the semimajor axis and period of the orbit. When spectral lines from one star are observed, the period is found from the period of the Doppler shifts; the magnitude of the shifts yields the stellar velocity times the sine of the inclination angle. The size of the orbit can thus be inferred. Individual masses can be determined in cases where two sets of spectral lines appear and the inclination angle can also be established. The quantity $M_\odot = 1.989 \times 10^{33}$ g is the solar mass. A correlation (MS) exists between the stellar mass and luminosity, with $L \propto M^n$, where $n \simeq 3$.

Hydrostatic equilibrium

$$\frac{dP}{dr} = -\frac{Gm(r)\rho(r)}{r^2}$$

and the classical perfect gas law implies that a star's central pressure P_c , density ρ_c and temperature T_c are related by

$$P_c \sim \frac{GM\rho_c}{R}, \quad P_c \sim \rho_c T_c,$$

so that $T_c \sim GM/R$. Here M and R are the total mass and radius of the star.

Energy is transported through a star like the Sun mostly through radiative transfer. An important physical parameter in this regard is the radiative opacity κ , or resistance to radiation flow, of matter. The luminosity $L(r)$ at any point is determined by the gradient of the radiation energy density

$$L(r) = \frac{16\pi a c r^2 T(r)^3}{3\kappa(r)\rho(r)} \frac{dT(r)}{dr}.$$

As a first approximation, we can treat the opacity as a constant. Dimensionally, this and the blackbody formula gives

$$L \propto R^2 T_{eff}^4 \propto R^4 T_c^4 / M \propto M^3.$$

Thus, $T_{eff} \propto T_c R^{1/2} / M^{1/4} \propto M^{3/4} R^{1/4} \propto (RT_c)^{3/4}$.

On the other hand, the energy generation rate for the H burning reactions varies roughly as $\rho^2 T^5$, which means that integrated over the stellar volume

$$L \sim \int_0^R \rho^2 T^5 dr^3 \sim \rho_c^2 T_c^5 R^3 \sim M^7 R^{-8}.$$

These relations combined imply that $R \sim M^{1/2}$.

Radii of stars are more difficult to measure. Interferometric techniques have yielded sizes of some stars, but otherwise size estimates rely on the Planck blackbody formula $L = 4\pi R^2 \sigma T_{eff}^4$. However, stars are not blackbodies, and stellar atmosphere models must be constructed to determine reliable effective temperatures. The radius of the Sun is $R_\odot = 6.96 \times 10^{10}$ cm. Radii of main sequence stars range from 0.3 – 20 times that of the Sun, but red giants like Betelgeuse have radii greater than 300 R_\odot , and white dwarfs like Sirius B have radii of order 0.02 R_\odot .

6. Stellar Energetics

The order of magnitude binding energy of the Sun is

$$\Omega = -GM^2/R = -4 \times 10^{48} \text{ erg.}$$

If this energy was liberated at its present rate, the sun would live for a time

$$\tau = -\Omega/L \approx 10^{15} \text{ s} = 3 \times 10^7 \text{ yr.}$$

It is known, however, that the Sun is 150 times older. In fact, the energy to power the Sun comes from nuclear reactions converting H into He, which convert a fraction of its rest mass into energy. This is now observationally established because of the detection of neutrinos, produced when neutrons are converted to protons, from the Sun. The total rest mass energy of the Sun is

$$M_\odot c^2 \simeq 2 \times 10^{54} \text{ erg}$$

which is enough to power the Sun for 5×10^{20} s or 1.6×10^{13} yr. In fact, nuclear reactions in the Sun only convert 0.007 of the rest mass into energy, and only the inner 10% of the Sun will convert its H to He. Thus the estimated solar lifetime is 1.1×10^{10} yrs.

The lifetimes of other stars can be estimated from $L \sim M^3$: The available energy in a star is proportional to M , so the stellar lifetime $\tau \sim M/L \sim M^{-2}$.

The estimate $T_c \approx GM/R$ leads to the estimate that, for the Sun, $T_c \simeq 10^7$ K. Therefore, the average thermal energy of protons at the solar center is

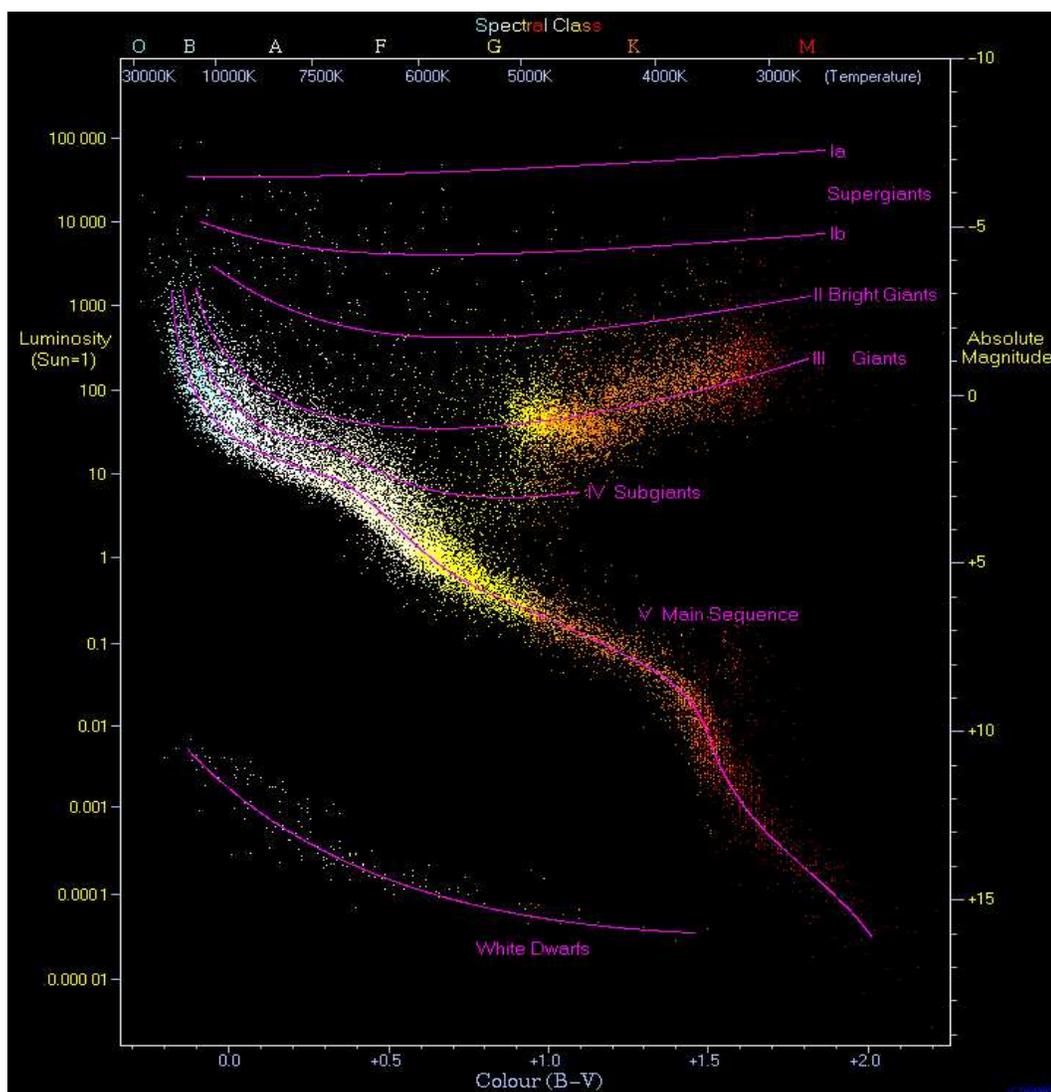
$$k_B T = 8.62 \times 10^{-8} T \text{ keV/K} \simeq 1 \text{ keV}.$$

But the Coulomb barrier between two positively charged protons is

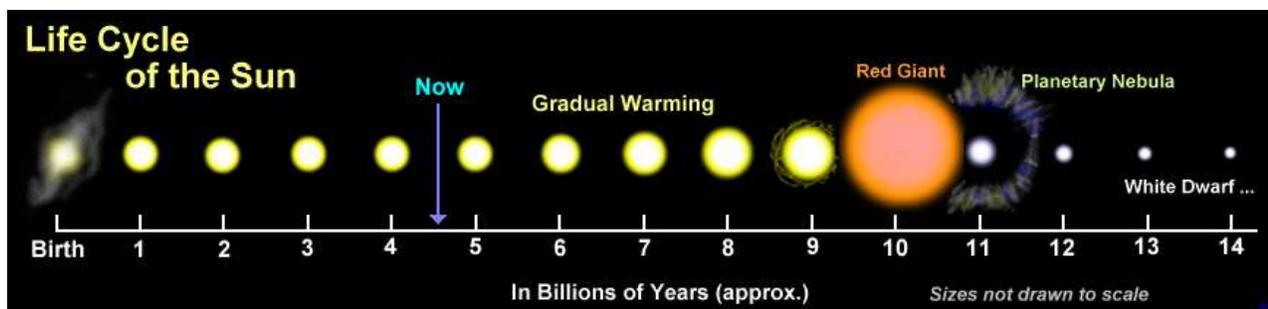
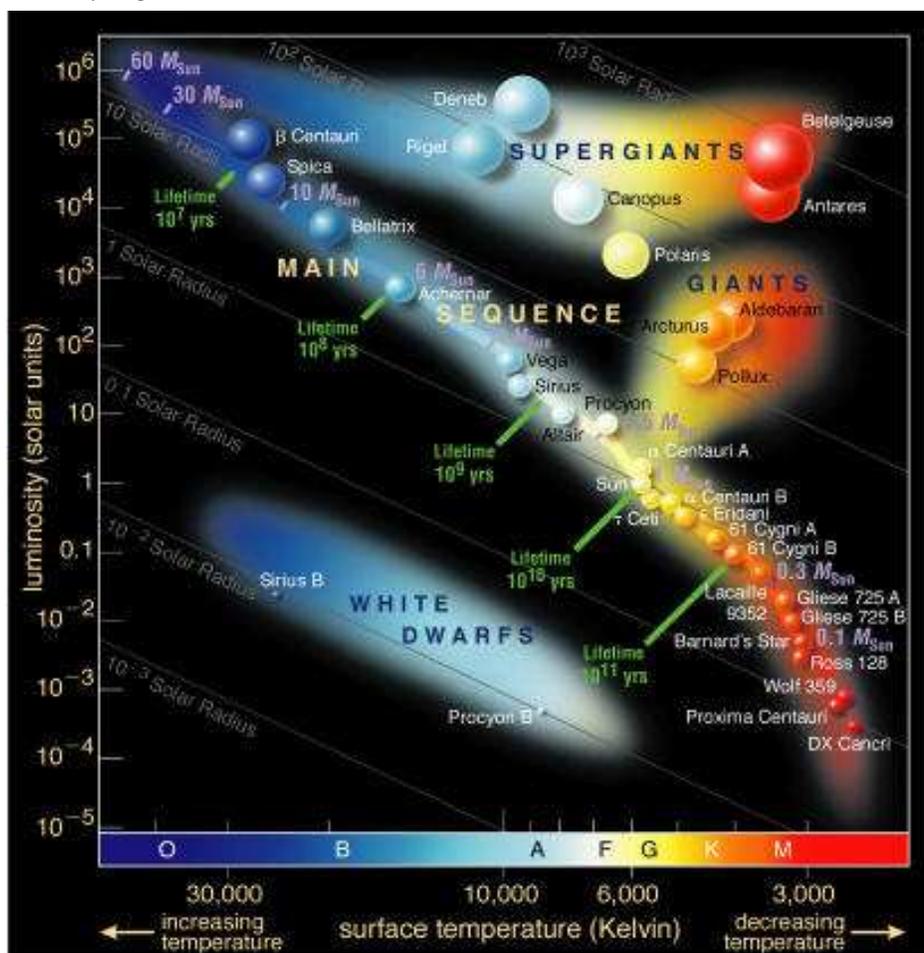
$$V = \frac{e^2}{r} = \frac{1.44 \text{ MeV} \cdot \text{fm}}{r}.$$

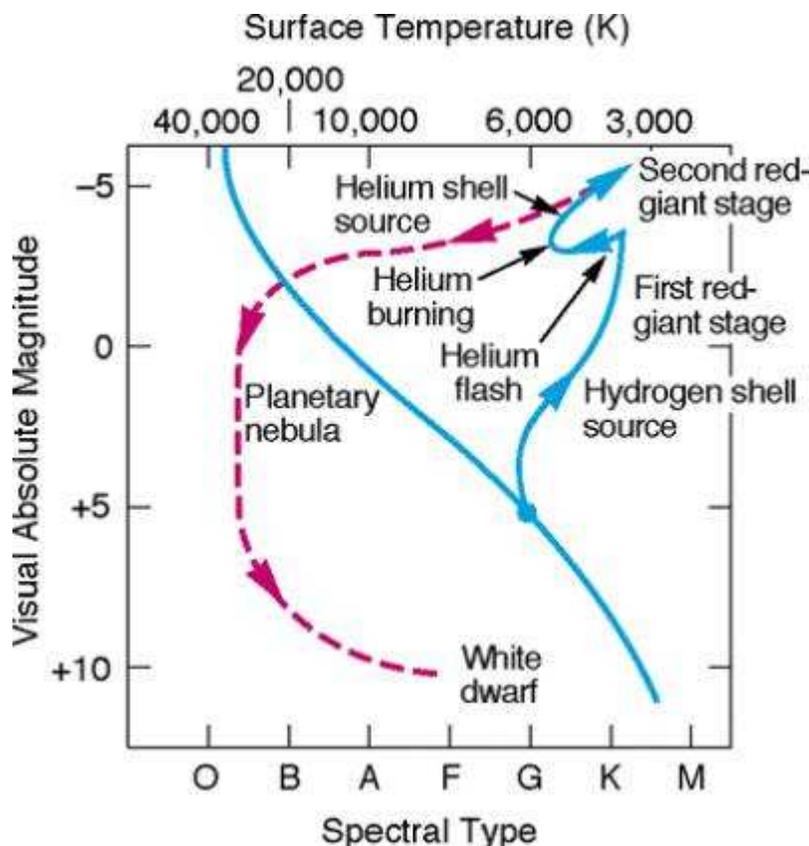
Since the proton radius is of order 1 fm, how can protons of energy 1 keV penetrate a 1 MeV barrier? The answer is by quantum mechanical tunneling.

7. The Hertzsprung-Russel Diagram



Astronomers discovered at the beginning of the twentieth century that plotting the observed luminosities of stars versus their temperatures or colors yielded highly significant correlations. Most stars lie on a narrow swath known as the Main Sequence, going from high L and high T to small L and small T . These stars burn H into He. Another group of stars has much greater L for the T than main sequence stars; these are generally known as red giants, and obviously have very large radii (up to 10000 R_{\odot}). There is another group with the opposite tendency; these have very small radii (of order Earth-like radii, 0.001 R_{\odot}), and are called white dwarfs. These different groupings represent stars at different points in their evolution. Red giants are stars near the end of their lives, and white dwarfs are their final dying embers.





The evolution of the Sun is shown schematically in Fig. 7.3. The red giant phase occurs after the interior of the Sun is exhausted of hydrogen and helium burning initiates. The Sun is not massive enough to burn elements beyond He, so after shedding roughly half its mass in a violent wind leading to a planetary nebula, the core slowly cools into a white dwarf remnant composed largely of carbon and oxygen.

Stellar evolution can be easily observed using the HR diagram. Clusters contain stars presumably formed at nearly the same time and all stars within the cluster are at nearly the same distance from the Sun. The most massive and luminous stars have relatively short lifetimes, so they should move off the main sequence first. Once the luminosity of one star in the cluster is determined, the luminosities of all the stars are then known. The observed “turn-off” luminosity provides an age for the cluster, which is the main-sequence lifetime of a star whose mass has that luminosity.

There are two different types of stellar clusters: galactic clusters which are loose associations of hundreds to thousands of stars, and globular clusters which are tight groupings of hundreds of thousands of stars. Galactic clusters are relatively young, with ages ranging from millions to billions of years, but globular clusters are uniformly about 10 billion years old. Chemical compositions of cluster stars are also different: galactic cluster stars have abundances similar to that of the Sun, in which the mass fraction of elements heavier than He is a few percent, but stars in globular clusters have heavy element mass fractions of hundredths of a percent. The difference in galactic distributions, abundances and motions of the two types of clusters has resulted in the identification of two major populations of stars: Type I, stars formed recently like the Sun, and Type II, stars formed when the Galaxy was formed or even before.

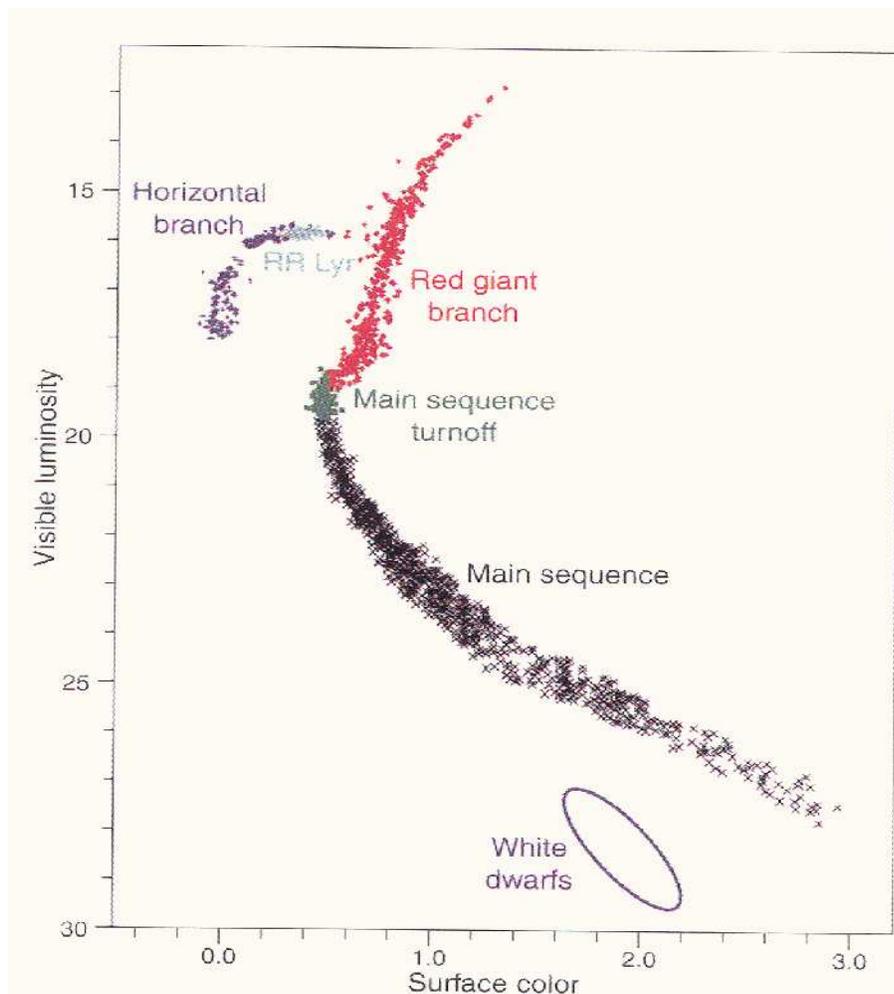


Fig. 2. A schematic color-magnitude diagram for a typical globular cluster (33) showing the location of the principal stellar evolutionary sequences. This diagram plots the visible luminosity of the star (measured in magnitudes) as a function of the surface color of the star (measured in B-V magnitude). Hydrogen-burning stars on the main sequence eventually exhaust the hydrogen in their cores (main sequence turnoff). After this, stars generate energy through hydrogen fusion in a shell surrounding an inert hydrogen core. The surface of the star expands and cools (red giant branch). Eventually the helium core becomes so hot and dense that the star ignites helium fusion in its core (horizontal branch). A subclass is unstable to radial pulsations (RR Lyrae). When a typical globular cluster star exhausts its supply of helium, and fusion processes cease, it evolves to become a white dwarf.

8. Stellar Evolution and Nucleosynthesis

The chemical changes in a star are what produce stellar evolution. Stars with a homogeneous composition, as a function of mass, will occupy a narrow band in the H-R diagram strikingly similar to the main sequence. Stars with an abrupt chemical inhomogeneity can form a different sequence, one that occupies the red giant region. This occurs when about 10% of the star has a heavy core. The burning of H to He slowly generates the heavy core. Observations are well-fit by structures with both chemical inhomogeneities and a degenerate or partially degenerate core/non-degenerate envelope configuration. Some of these structures are prone to pulsational instabilities, observed as regular variabilities in temperature and luminosity. Finally, completely degenerate stellar structures form a third

sequence that resembles the white dwarf branch. Therefore, stellar structure has to be understood before stellar evolution can be discussed.

Nucleosynthesis is the natural by-product of stellar evolution. The most bound nucleus is Fe, and slowly nuclear reactions in stars strive toward nuclei with the greatest binding. For the most part, however, the heavy elements produced in stars are trapped within unless the star can be disrupted. Low-mass stars, unless they are in close binaries, have little opportunity of disruption. However, high mass stars, and low-mass stars in close binaries, can undergo supernova explosions in which the star is partially or totally disrupted, accompanied by a tremendous release of energy: in photonic emissions and kinetic energy, and, in some cases, neutrinos. Supernovae involving white dwarfs (products of low-mass stellar evolution) in binaries are thermonuclear detonations which totally expell all the stellar material, while gravitational collapse of the cores of massive stars lead to neutron star (and black hole) formation accompanied by the expulsion of the star's massive envelope. These explosions also provide a natural method of enriching the chemical composition of the interstellar medium with heavy elements, leading to galactic chemical evolution. The pathways for creating elements heavier than Fe, which are less bound again, has to go through an extensive neutron capture environment whose site is still debated. It could be in gravitational collapse supernovae, in the envelopes of certain red giants, or in the breakup of neutron stars themselves.

