

Galactic Chemical Evolution

Ingredients The modeling of galactic chemical evolution requires four ingredients:

- Initial conditions: is the chemical composition of the initial gas primordial or pre-enriched? Is the system closed or open (with infall and outflow)?
- The stellar birthrate function $B(m, t)$, which is the rate at which gas is turned into stars of a given mass. It is often expressed as the product of the star formation rate (SFR) and the initial mass function (IMF). There are reasons for believing that the IMF is independent of time, in which case we can write

$$B(m, t) = \text{SFR} \times \text{IMF} = \psi(t) \phi(m).$$

- Stellar properties: stellar lifetimes τ_m , remnant masses w_m and stellar yields $Y_i(m)$, or how elements are produced in stars and restored into the ISM.
- Gas flows, i.e., infall, outflow or radial flows.

Some stellar properties The lifetime of a star of mass m (in M_\odot) with solar metallicity (Z_\odot) is approximately

$$\tau_m \simeq 11.3m^{-3} + 0.06m^{-.75} + 0.0012 \text{ Gyr}.$$

Stars of lower metallicity live shorter lives if their initial masses are less than about $10 M_\odot$; longer otherwise.

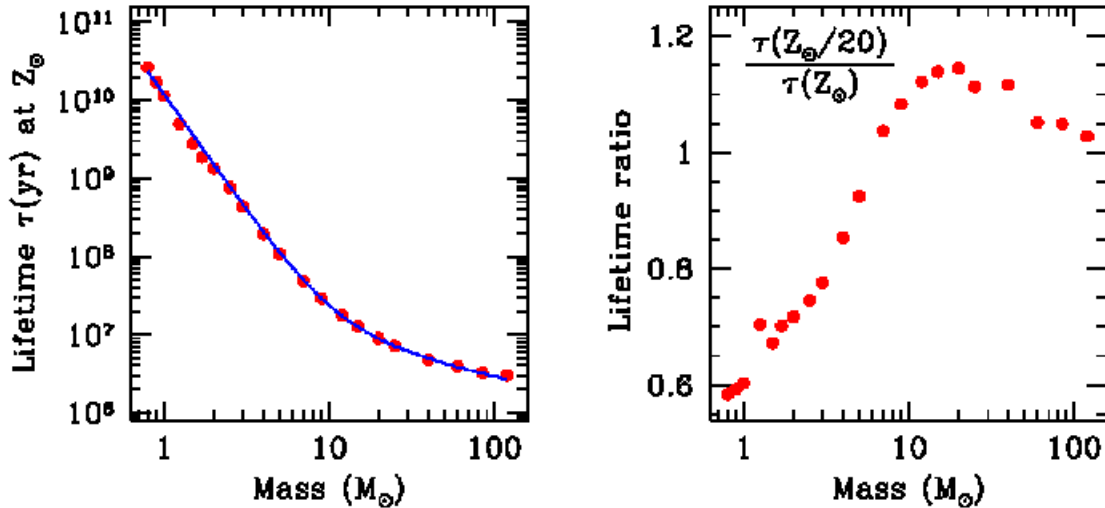


Figure 1: *Left:* Lifetimes of stars of Z_\odot . Blue curve is a fit. *Right:* Ratio of stellar lifetimes $Z_\odot/20:Z_\odot$. (From Geneva stellar models).

For low- and intermediate-mass stars, the remnant is a white dwarf with $w_m \simeq 0.08m + 0.47$; for stars $9 < m < 25$ the remnant is likely a neutron star with $w_m \simeq 1.35$; for larger masses the remnant appears to be a black hole with $w_m \simeq 0.24m - 4$.

Observational constraints

- The age-metallicity $t-Z$ relation, or AMR. This has been obtained by combining metallicity measurements with stellar ages derived from theoretical isochrones. The result can be expressed in a number of ways, given the errors in the observations:

$$\begin{aligned} \log_{10} \frac{t}{1 \text{ Gy}} &= 0.93 + 1.3 [\text{Fe}/\text{H}] - 0.04 [\text{Fe}/\text{H}]^2 \\ [\text{Fe}/\text{H}] &= 0.68 - \frac{11.2 \text{ Gy}}{t + 8 \text{ Gy}}, \end{aligned} \quad (1)$$

where $[\text{Fe}/\text{H}] \equiv \log_{10}(Z/Z_{\odot})$. The first form is more useful when the initial metallicity is very small, but it does not have an effective upper limit. As $t \rightarrow \infty$, we expect that gas will tend to be continuously exhausted which raises Z to a terminal value, as in the second form. The second form has $Z(t \rightarrow \infty)/Z_{\odot} \approx 4.8$, $Z(t_1)/Z_{\odot} \approx 1.32$ at the present, and $Z(0)/Z_{\odot} \approx 0.19$ at $t = 0$. Interestingly, when one plots the second form in a linear-linear plot, one finds that it is well-approximated by a linear relation

$$Z = Z(0) + [Z(t_1) - Z(0)] \frac{t}{t_1}, \quad (2)$$

where $t_1 \approx 12 \text{ Gy}$ is the time since disk formation.

- The present-time surface gas density $\sigma_g = 13 \pm 3 \text{ M}_{\odot} \text{ pc}^{-2}$
- The present-time surface star density $\sigma_* = 43 \pm 5 \text{ M}_{\odot} \text{ pc}^{-2}$
- The present-time SFR $\psi_0 = 2 - 5 \text{ M}_{\odot} \text{ pc}^{-2} \text{ Gyr}^{-1}$
- The present-time infall rate $0.3 - 1.5 \text{ M}_{\odot} \text{ pc}^{-2} \text{ Gyr}^{-1}$
- The present-time mass function
- Solar abundances
- Observed $[X_i/\text{Fe}]$ vs. $[\text{Fe}/\text{H}]$ relations
- Observed G-dwarf metallicity distribution
- Average SNIa ($30 \pm 20 \text{ yr}^{-1}$) and SNII ($120 \pm 80 \text{ yr}^{-1}$) rates

Chemical evolution equations

Theoretically, we can attempt to calculate the evolution of Z in the solar neighborhood. Defining the birthrate B of stars as the mass of stars born per unit time, the total stellar birthrate is:

$$\psi(t) = \int_{m_L}^{\infty} mB(m, t) dm.$$

m_L is the lower mass limit for stars, usually taken to be $0.1 M_{\odot}$. If the initial mass function (IMF) is constant in time it is $\phi(m) = B(m, t)/\psi(t)$ and is normalized

$$\int_{m_L}^{\infty} m\phi(m) dm = 1.$$

The total mass M and the mass of gas m_g are usually defined in terms of a mass per unit area integrated vertically through the galactic disc. The gas mass changes because of star formation (ψ), gas loss from stars $R\psi$, and inflow (f) and outflow (o):

$$\frac{dm_g}{dt} = -\psi(t) + E(t) + f(t) - o(t).$$

E is the rate of mass ejection. If τ_m is the lifetime of a star with mass m , which sheds at death all but a remnant mass w_m , and if $m(t)$ is the mass for which $\tau_m = t$, we have

$$E(t) = \int_{m(t)}^{\infty} (m - w_m) \phi(m) \psi(t - \tau_m) dm$$

For a specific chemical element,

$$\frac{d(m_g X_i)}{dt} = -\psi X_i + E_i(t) + X_{i,f} f(t) - X_{i,o} o(t).$$

Usually, $X_{i,o} = X_i$, but if hot supernova ejecta (rich in metals) leaves the system, then $X_{i,o} > X_i$. The rate of ejection of element i is

$$E_i(t) = \int_{m(t)}^{\infty} Y_i(m) \phi(m) \psi(t - \tau_m) dm$$

where Y_i is the stellar yield of element i .

Derivation of the IMF

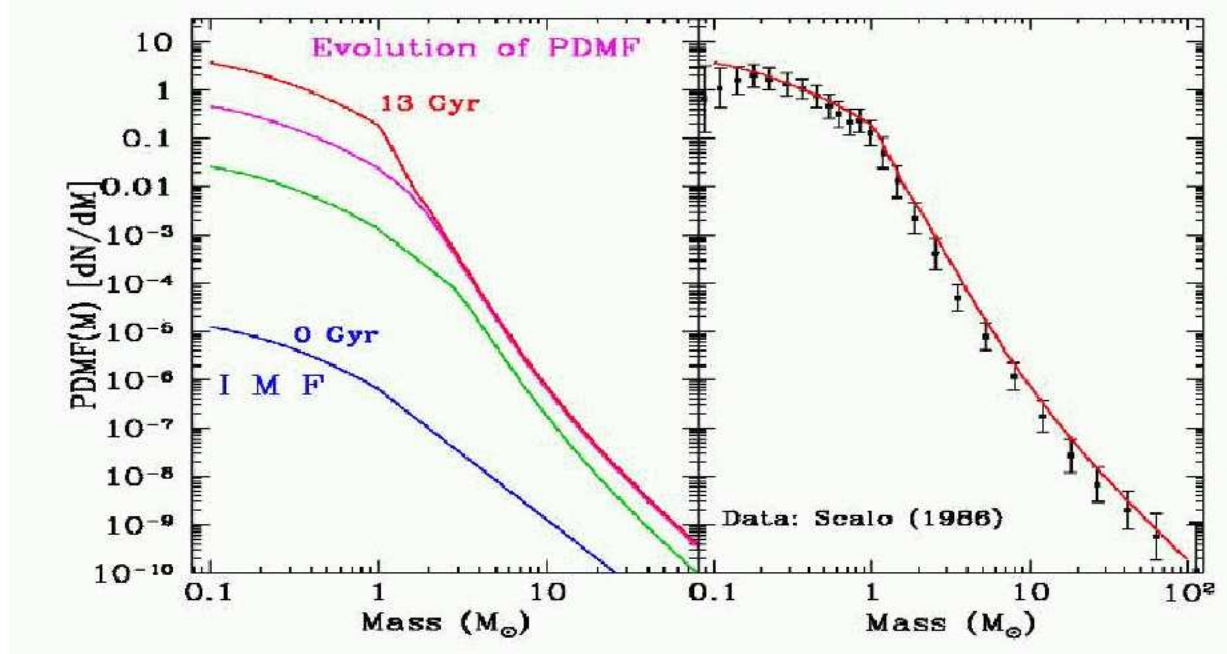


Figure 2: The present-day mass function in differential form $dN(m)/dm$. *Left:* The evolution of the present-day mass function. At $t = 0$ it is equal to the IMF $\phi(m)$. The final one ($t = 13$ Gyr) is the presently observed one. *Right:* The present-day mass function compared with data from Scalo.

The present-day mass function $N(m)$, for stars with initial masses $0.1 < m < 1$ which have lifetimes $\tau_m > t_G = 14$ Gyr (the Hubble time),

$$N(m) = \int_0^{t_G} \phi(m) \psi(t) dt.$$

If the IMF is constant in time,

$$N(m) = \phi(m) \langle \psi \rangle t_G,$$

where $\langle \psi \rangle$ represents the average SFR in the past. For stars with initial masses $2 < m < \infty$, $\tau_m \ll t_G$,

$$N(m) = \int_{t_G - \tau_m}^{t_G} \phi(m) \psi(t) dt \simeq \phi(m) \psi_0 \tau_m,$$

where $\psi_0 = \psi(t_G)$ and we assumed $\psi(t)$ didn't change during the interval $t_G - \tau_m$ and t_G . For $1 < m < 2$ we cannot derive the IMF, but can use continuity to bridge the gap.

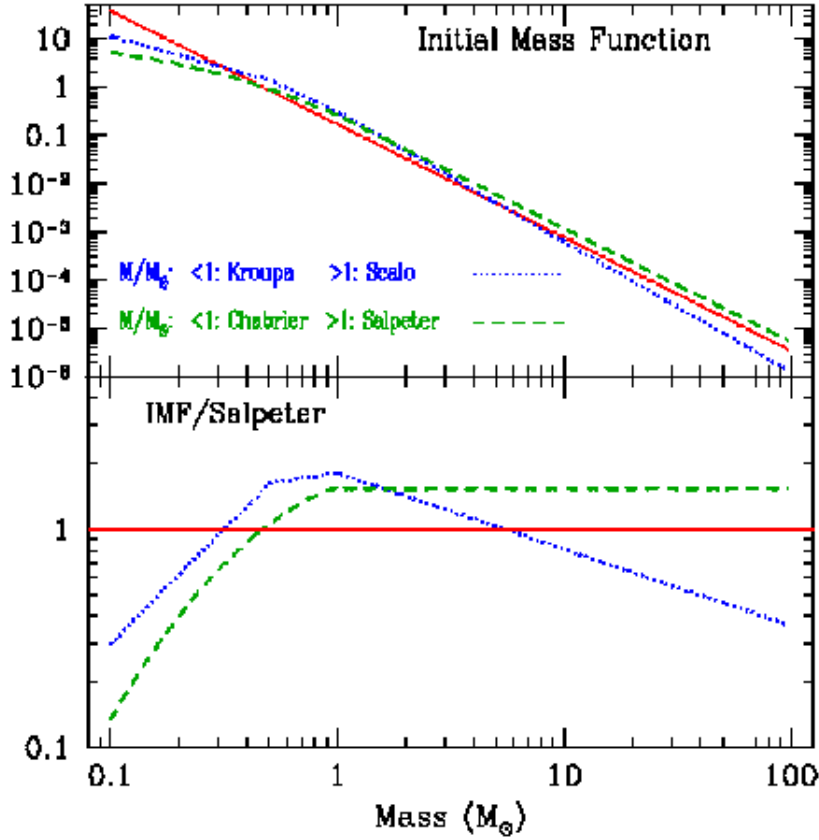


Figure 3: *Top:* Initial mass functions ($\phi(m)$). Red curve is $\phi_{Salpeter}$. *Lower:* Ratio of mass functions to $\phi_{Salpeter}$.

Based on stars in the solar neighborhood and accounting for various biases but not stellar multiplicity, Salpeter found

$$\phi_{Salpeter} = \frac{dN}{dm} = Am^{-1-x}, \quad m > 0.1$$

where $x = 1.35$. The normalization of ϕ , using $m_L = 0.1 M_\odot$, implies that $A = 0.35 \cdot 10^{-0.35} \simeq 0.156$. More recent studies by Scalo; Chabrier; Kroupa; and Reid and Gizis find fewer stars in the low-mass range. This is illustrated in Fig. 3.

One can define the return mass fraction $R(t)$ to be

$$R(t) = \int_{m(t)}^{\infty} (m - w_m) \phi(m) dm;$$

it is the fraction of the mass of a stellar generation that returns to the ISM. For the IMF's in the figure, $R_{Salpeter}(\tau_1) = 0.278$, $R_{Kroupa}(\tau_1) = 0.285$ and $R_{Chabrier}(\tau_1) = 0.34$. For comparison, $R_{Salpeter}(\tau_9)$ and $R_{Salpeter}(\tau_{25})$ are 0.17 and 0.12, respectively.

Recipes for the SFR

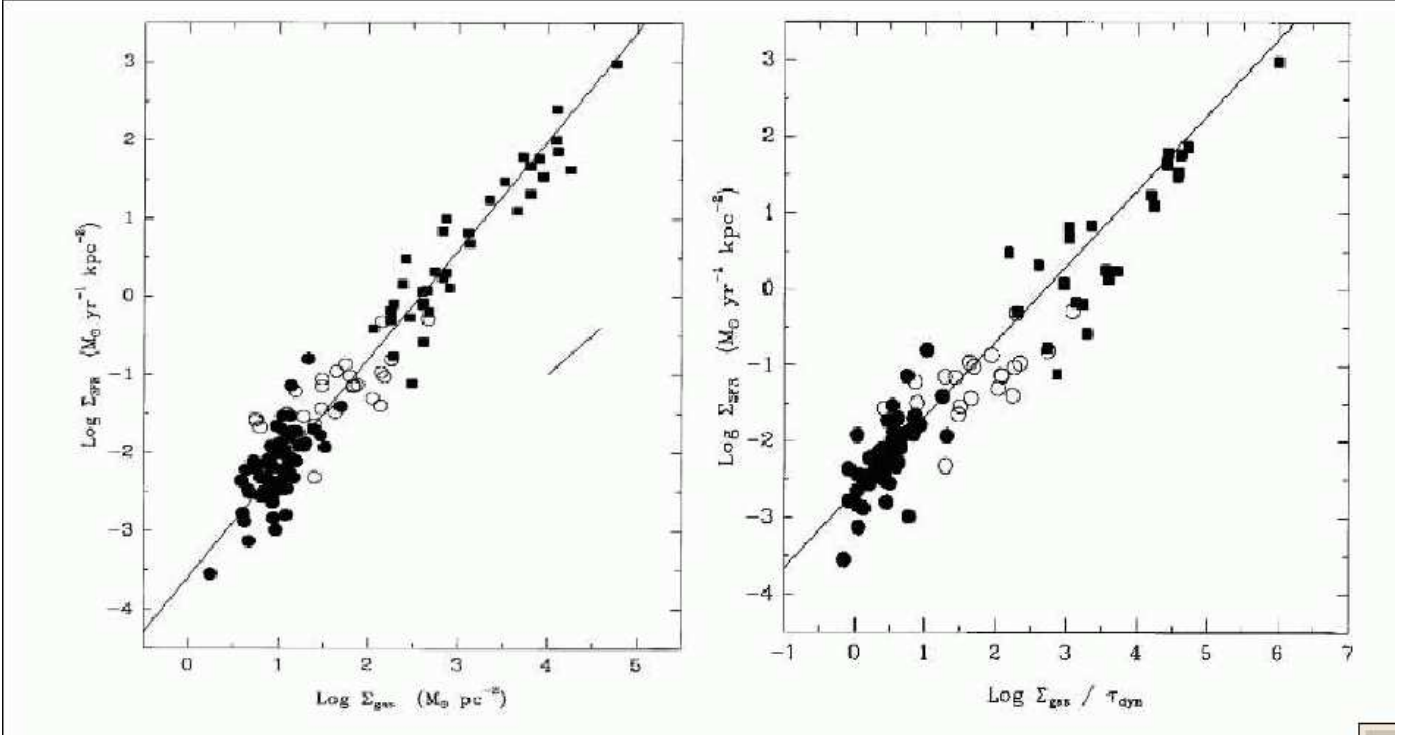


Figure 4: *Left:* Average surface density of star formation rate (ψ) as a function of average gas (HI + H₂) surface density (Σ_{gas}). The solid curve is a power law with exponent 1.4. *(Right:)* Average surface density of star formation rate as a function of Σ_{gas}/τ_{dyn} . The solid curve is a power law with exponent of 1.0.

The SFR is probably the most uncertain quantity in galactic chemical evolution models. Most observational information, such as from luminous star counting, flux measurements in recombination lines from gas ionized by OB stars, the UV flux of OB stars, far-infrared emission of dust heated by UV from nearby hot stars, and surface density of supernova remnants and pulsars, concerns the rate for $m > 2$. There is also no theory capable of predicting the large-scale SFR given the various physical ingredients (gas density, relative gas/star masses, temperature, composition, magnetic fields, molecular cloud collision frequency, galactic rotation, etc.). Schmidt suggested

$$\psi = \nu m_g^N$$

but it is not clear if m_g should be a surface or a volume density. Kennicutt finds a good correlation between SFR and total (atomic + molecular) gas,

and if m_g is a surface density, $N \approx 1.4$. But one could also fit the same data with the expression

$$\psi \propto m_g / \tau_{dyn}$$

where τ_{dyn} is the dynamical timescale related to galactic rotation $\tau_{dyn} = R/V(R)$.

Analytical solutions with the IRA

The instantaneous recycling approximation (IRA), introduced by Schmidt, is often assumed. Stars are either eternal ($\tau_m \gg t_G$) or “dead at birth” ($\tau_m \ll t_G$). For practical purposes, the dividing line is about $1 M_\odot$ corresponding to $t_G \simeq 12$ Gyr. Assuming IRA allows one to replace $\psi(t - \tau_m)$ with $\psi(t)$ and removing ψ from the mass integrals. In addition, the return function $R(t)$ is then set to a constant $R \equiv R(t_G)$, which is of order 0.3. Solutions involve the yield quantity p_i of a given nuclide, defined by

$$p_i = \frac{1}{1 - R} \int_{m(t_G)}^{\infty} y_i(m) \phi(m) dm.$$

This is the newly created amount of nuclide i by a stellar generation, per unit mass of stars locked into eternal objects. The net yield of nuclide i is

$$y_i(m) = Y_i(m) - X_i(m - w_m),$$

and $1 - R$ is the mass locked up in low-mass stars and compact object remnants.

One now finds

$$E(t) \simeq \psi(t) \int_{m(t_G)}^{\infty} (m - w_m) \phi(m) dm \equiv R\psi(t)$$

and

$$E_i(t) \simeq \psi(t) \int_{m(t_G)}^{\infty} [y_i + X_i(m - w_m)] \phi(m) dm = [X_i R + (1 - R) p_i] \psi(t).$$

Neglecting inflow and outflow for the moment, usually called the Closed-Box approximation, one finds

$$\begin{aligned} \frac{dm_g}{dt} &= - (1 - R) \psi(t), \\ \frac{dX_i m_g}{dt} &= - X_i \psi(t) + [X_i R + (1 - R) p_i] \psi(t), \\ \frac{dM}{dt} &= 0. \end{aligned}$$

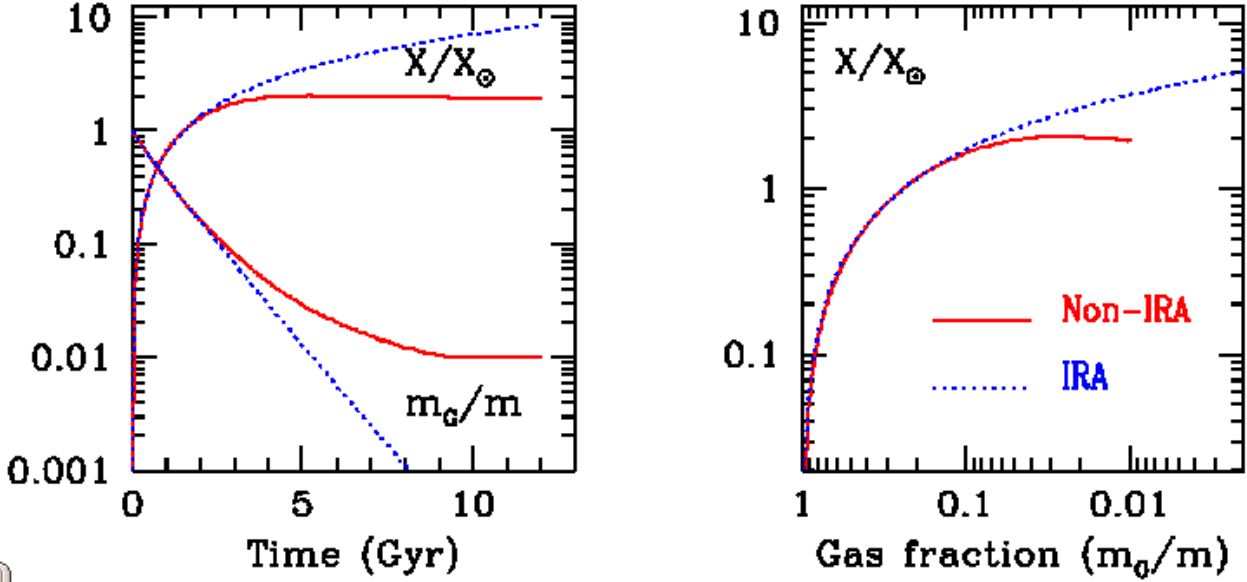


Figure 5: Closed-box model with (dotted curves) and without (solid curves) IRA. *Left:* Metallicity and gas fraction as a function of time. *Right:* Metallicity as a function of gas fraction.

These equations can be manipulated to yield

$$m_g \frac{dX_i}{dt} = -p_i \frac{dm_g}{dt}, \quad X_i = X_{i,0} + p_i \ln \frac{M}{m_g},$$

where $X_{i,0}$ is the initial abundance. If a Schmidt Law of the form $\psi = \nu m_g$, with ν a constant, is assumed, one finds

$$m_g = e^{-\nu(1-R)t}, \quad X_i = X_{i,0} + p_i \nu (1-R)t.$$

The Closed Box model must be constrained to produce the observed metallicity in the required time. The observed gas fraction is $m_g/M \simeq 0.2$ today. With $Z_i = 0$, $R \simeq 0.3$, $m_g(t_1)/M \simeq 0.2$ and $Z(t_1 - 4.5) = 1$, where the present time is $t_1 = 12$ Gyr, values for $\nu \simeq 0.19$ Gyr $^{-1}$ and $p \simeq 0.99$ are implied. It also predicts that the cumulative metallicity distribution, the number of stars with metallicity lower than Z as a function of Z , is

$$N_*(Z < Z') = \int_0^{t'} \psi(t) dt = \frac{1}{1-R} [M - m_g(t')], \quad Z(t') = Z'$$

or

$$N_*(Z < Z') = \frac{M}{1-R} \left(1 - \exp \left[\frac{Z_i - Z'}{p} \right] \right),$$

where $Z_1 \simeq 1.6$ is the present-day metallicity. This function is proportional to $Z' - Z_i$ when the argument of the exponential is small, and consequently overpredicts the number of metal-deficient G-K dwarf stars if $Z_i = 0$.

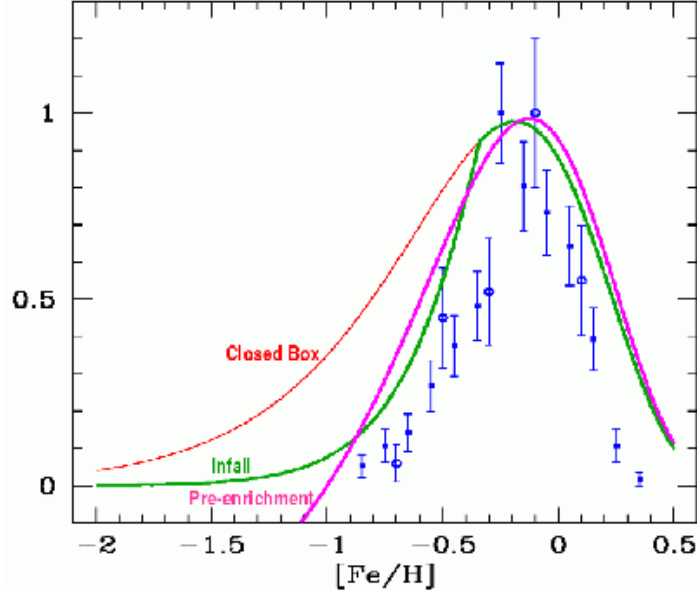


Figure 6: Metallicity distribution $\frac{d[N(Z < Z')/N(Z < Z_1)]}{d \log Z}$ of long-lived stars for a Closed Box model, an exponentially decreasing infall (timescale 7 Gyr) model and a model with pre-enrichment ($X_{i,0} = 0.08X_\odot$) compared to data for the solar neighborhood.

Another way of expressing this information is to look at the differential metallicity distribution for the Closed Box model

$$\frac{d[N(Z < Z')/N(Z < Z_1)]}{d \log Z} = \frac{\ln(10)}{1 - e^{(Z_i - Z_1)/p}} \frac{Z' - Z_i}{p} e^{(Z_i - Z')/p}.$$

This function has a peak when $Z' = p + Z_i$ and must be 0 when $Z' \leq Z_i$. It is shown in Fig. 6 for the cases $Z_i = 0$ and $Z_i = 0.08Z_\odot$, respectively, and using $p = 0.65$. Although the model with pre-enrichment apparently fits the data, it is hard to justify because the halo mass is much smaller than the disk mass. Even though the halo's maximum metallicity is about $0.1Z_\odot$, the average halo metallicity is a third of this and the halo mass is 20 times smaller than the disk: the disk cannot have had such large pre-enrichments.

In the case of flows, analytic solutions can be found for some simple cases, but that might be too restrictive. In any case, they can be instructive. For

example, in the case of a outflow proportional to the SFR, $o = k\psi$, one finds

$$X_i = X_{i,0} + \frac{p_i}{1+k} \ln \frac{M}{m_g}.$$

In the case of nuclides that are secondary, such as s-process elements that depend on pre-existing heavy elements for their formation, the yield $p_{secondary} = \alpha X_{primary}$ and its evolution with $X_{secondary,0} = 0$ is

$$X_{secondary} = \alpha X_{primary} \ln \frac{M}{m_g} = \frac{\alpha}{p_{primary}} X_{primary}^2.$$

The abundance of secondaries therefore grows much faster than primaries.

Models with infall

Another possibility is to consider models with an appreciable inflow of metal-deficient gas into the disc up to the present time. A standard way of modelling this is to suppose that the inflowing matter has a metallicity Z_i and an infall rate $f = \Lambda(1-R)\psi$, i.e., it is proportional to the star formation rate. In general, $\Lambda \leq 1$; the case $\Lambda = 1$ is an extreme case in which the gas mass of the galaxy remains constant in time. The solution in the case that $Z_i = 0$ becomes

$$\frac{m_g(t)}{m_g(0)} = (1 - Z\Lambda/p)^{\Lambda^{-1}-1},$$

which implies that the gas fraction tends to zero when $Z = p/\Lambda$. Therefore, the value of Λ can be constrained by the present metallicity and gas fraction. But it is clear from the case with $\Lambda = 0$ that $p = 0.81$ predicts $Z_1 = 1.3$, so a consistent solution can only be found if $p \geq 0.81$: for example, $p = 0.85$ suggests that $\Lambda = 0.16$ and $p = 0.95$ implies $\Lambda = 0.44$. Thus, it would appear with this model that outflow, rather than inflow, would better fit the data.

A convenient way of expressing the solution is in terms of the variable

$$\mu(t) = \frac{M(t) - M(0)}{M(0)} = \frac{m_g(t) - M(0)}{M(0)} \frac{\Lambda}{\Lambda - 1} = \Lambda \frac{1 - R}{M(0)} \int_0^t \psi(t) dt.$$

$$Z = Z_i + \frac{p}{\Lambda} \left[1 - \left(1 - \frac{1 - \Lambda}{\Lambda} \mu \right)^{\Lambda/(1-\Lambda)} \right], \quad \Lambda \neq 1$$

$$Z = Z_i + p [1 - e^{-\mu}]. \quad \Lambda = 1$$

The cumulative metallicity distribution in this model is

$$N(Z < Z') = \int_0^{t'} \psi(t) dt = \frac{M(0)}{\Lambda(1-R)} \mu(t').$$

Thus

$$\begin{aligned} \frac{N(Z < Z')}{N(Z < Z_1)} &= \frac{\mu(t')}{\mu(t_1)} = \frac{1 - [1 - \Lambda(Z - Z_i)/p]^{\Lambda^{-1}-1}}{1 - [1 - \Lambda(Z_1 - Z_i)/p]^{\Lambda^{-1}-1}}, & \Lambda \neq 1 \\ &= \frac{\ln[1 + (Z_i - Z')/p]}{\ln[1 + (Z_i - Z_1)/p]}. & \Lambda = 1 \end{aligned}$$

The differential metallicity distributions are

$$\begin{aligned} \frac{dN(Z < Z')/N(Z < Z_1)}{d \log Z} &= \\ &= \ln(10) (1 - \Lambda) \frac{Z' - Z_i (1 - \Lambda(Z' - Z_i)/p)^{\Lambda^{-1}-2}}{p \left(1 - \left(1 - \frac{\Lambda(Z_1 - Z_i)}{p}\right)^{\Lambda^{-1}-1}\right)}, & \Lambda \neq 1 \\ &= -\ln(10) \frac{Z' - Z_i}{p + Z_i - Z'} \frac{1}{\ln(1 - (Z_1 - Z_i)/p)}. & \Lambda = 1 \end{aligned}$$

The maximum of the differential metallicity distribution occurs at $Z' = p/(1 + \Lambda)$ for $\Lambda \neq 1$. However, for $Z - Z_i > p/\Lambda$ the solution is imaginary. Therefore the extreme case $\Lambda = 1$ only makes sense for small Z ; it cannot be applied for $Z = Z_1$. Therefore, the extreme case, $\Lambda = 1$, cannot reproduce the observed distribution of G-K dwarfs either at large or small metallicities, but smaller Λ values can produce acceptable results. Nevertheless, the required values of $p > 0.81$ are inconsistent with stellar evolutionary predictions.

A somewhat different infall model is supported by observations: an exponentially decaying infall with $Z_i = 0$ and an infall rate $f(t) = Ae^{-t/\tau}$. The results of such a model are portrayed in Fig. 6. The function f is normalized to give the observed total column density in the solar neighborhood $\Sigma_T \simeq 50 M_\odot \text{ pc}^{-2}$:

$$\int_0^{t_1} f(t) dt = \Sigma_T, \quad A = \frac{\Sigma_T \tau^{-1}}{1 - e^{-(t_G/\tau)}},$$

with a galactic age of $t_1 \simeq 12$ Gyr. The model is constrained by the requirement that the galactic gas fraction at the present time is approximately 0.2 and that the present metallicity is $Z_1 \simeq 1.5$. This model can be rendered analytically if the star formation rate is taken to be a Schmidt law, $\psi = \nu m_g$. The differential equations governing this model are

$$\begin{aligned} \frac{dm_g}{dt} &= -(1-R)\nu m_g + Ae^{-t/\tau}, \\ \frac{dM}{dt} &= Ae^{-t/\tau}, \\ \frac{dZm_g}{dt} &= (p-Z)(1-R)\nu m_g. \end{aligned} \quad (3)$$

The solutions are given, using $\alpha = (1-R)\nu - 1/\tau$, by

$$\begin{aligned} m_g(t) &= \frac{A}{\alpha} e^{-t/\tau} \left(1 - e^{-\alpha t}\right), \\ M(t) &= A\tau \left(1 - e^{-t/\tau}\right), \\ Z(t) &= (1-R) \frac{p\nu}{\alpha} \frac{e^{\alpha t} - \alpha t - 1}{e^{\alpha t} - 1}, \\ N(Z < Z') &= \nu \int_0^{t'} m_g dt = \frac{A\nu}{\alpha} \left[\alpha + \frac{1}{\tau} e^{-t'/\tau} - (1-R)\nu e^{-(1-R)\nu t'} \right]. \end{aligned} \quad (4)$$

Assuming an infall timescale $\tau = 6.9$ Gyr, and constraining the present-day gas fraction gives $\nu = 0.37$. Constraining the solar metallicity gives $p = 1.2$. The differential metallicity distribution has a peak at approximately $Z = 0.4p$ in this model, with a maximum value of approximately 1.

These examples suggest the following:

1) There could have been an early burst of star formation involving an IMF different than today's. There are very few stars of $2M_\odot$ or less involved in this burst. This can be modelled, without infall, using the above by setting $Z_i \neq 0$. However, this tends to underpredict the numbers of stars of higher metallicities unless Z_i is set to an unphysically large value.

2) There has been an appreciable inflow of metal-deficient gas into the disc up to the present time.

3) Since the metallicity distribution of the halo stars peaks around $[\text{Fe}/\text{H}] = -1.6$, while the disc has a peak around $[\text{Fe}/\text{H}] = -0.2$, it has been shown that a model with outflow could be appropriate for the halo.

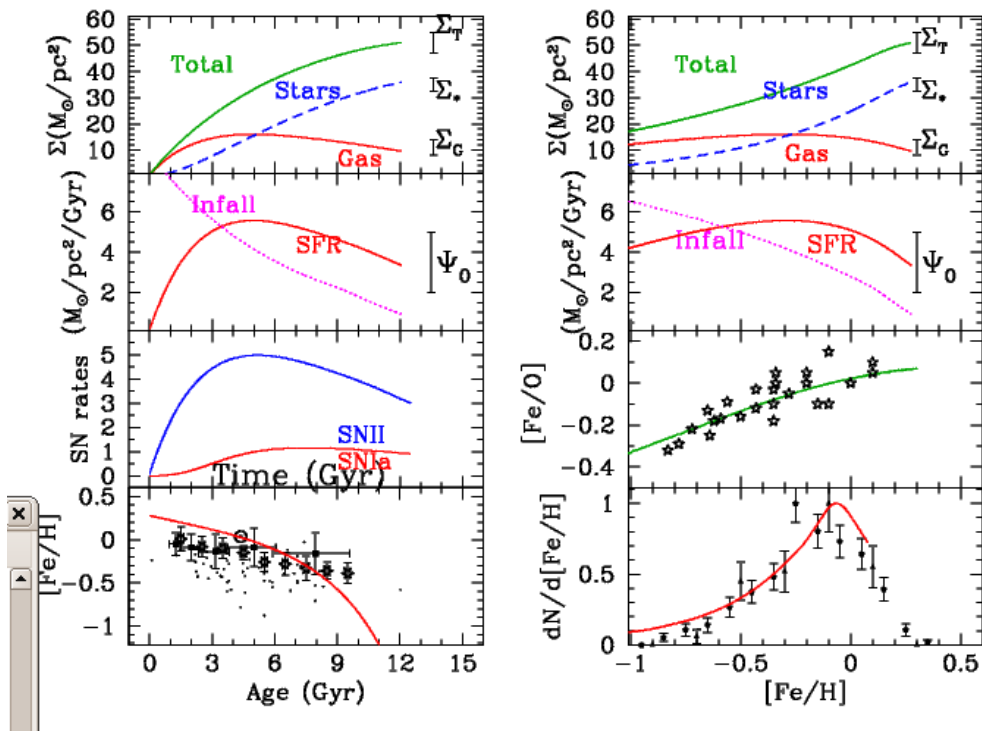


Figure 7:

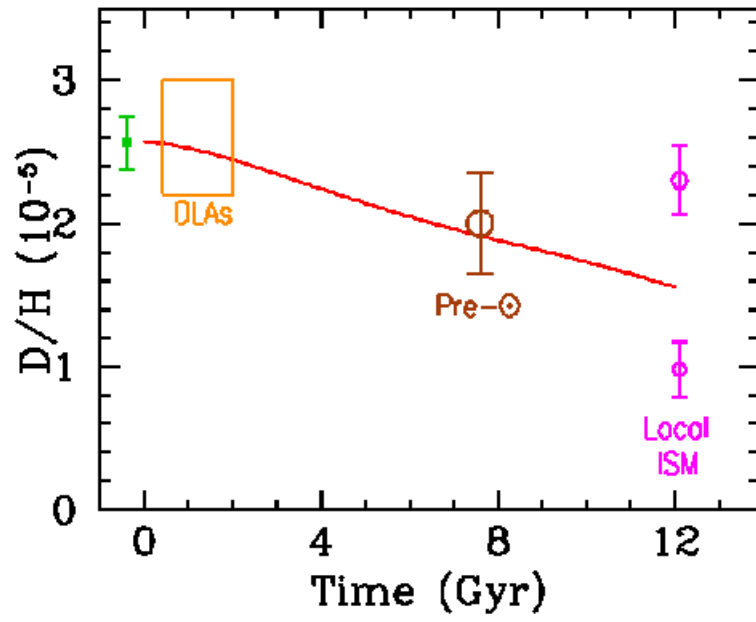


Figure 8: