Distances from Hipparcos trigonometric parallaxes

15,630 stars

$d < 100$ pc

Half of stars with $M_V > 10$ are not detected.
Local Stellar Luminosity Function

\[ \Phi(M) = \frac{\text{number stars } M_V \pm 1/2}{\text{volume over which seen}} \]

\[ \psi(M) = \phi_{MS}(M) \Theta \left( \frac{\tau_{\text{gal}}}{\tau_{MS}(M)} \right) \]

\[ \Theta(x) = 1 \text{ if } x \leq 0 \]
\[ \Theta(x) = x \text{ if } x > 0 \]

\[ \tau_{\text{gal}} \simeq 10 \text{ Gyr} \]

- Hipparcos
  - Reid et al. (2002)

Dim stars hard to find; Bright stars are rare.

Stars not uniformly distributed.

Stars in binaries mistaken for brighter single stars.
\( \xi(\mathcal{M}) \Delta(\mathcal{M}) = \text{number stars born with mass between} \ \mathcal{M} \ \text{and} \ \mathcal{M} + \Delta\mathcal{M} \)

Salpeter initial mass function \( \xi(\mathcal{M}) = \frac{\xi_0}{\mathcal{M}_\odot} \left( \frac{\mathcal{M}}{\mathcal{M}_\odot} \right)^{-2.35} \)

Total number: \( \int_{\mathcal{M}_\ell}^{\mathcal{M}_u} \xi(\mathcal{M}) d\mathcal{M} = \frac{\xi_0}{1.35} \left( \frac{\mathcal{M}_\odot}{\mathcal{M}_\ell} \right)^{1.35} \left[ 1 - \left( \frac{\mathcal{M}_\ell}{\mathcal{M}_u} \right)^{1.35} \right] \)

Total mass: \( \int_{\mathcal{M}_\ell}^{\mathcal{M}_u} \mathcal{M} \xi(\mathcal{M}) d\mathcal{M} = \frac{\xi_0 \mathcal{M}_\odot}{0.35} \left( \frac{\mathcal{M}_\odot}{\mathcal{M}_\ell} \right)^{0.35} \left[ 1 - \left( \frac{\mathcal{M}_\ell}{\mathcal{M}_u} \right)^{0.35} \right] \)

Total luminosity: \( \int_{\mathcal{M}_\ell}^{\mathcal{M}_u} L \xi(\mathcal{M}) d\mathcal{M} = \frac{\xi_0 L_\odot}{2.15} \left( \frac{\mathcal{M}_u}{\mathcal{M}_\odot} \right)^{2.15} \left[ 1 - \left( \frac{\mathcal{M}_\ell}{\mathcal{M}_u} \right)^{2.15} \right] \)
Distances From Motions

If $V_r$ and $V_t$ measured, and how they are related, we can find $d$.

$V_t = \mu d, \quad \mu(\text{arcsec/yr}) = V_t(\text{km/s})/(4.74d(\text{kpc}))$

Example: Distance to Galactic center, if orbit edge-on and perpendicular

$2a = (s/1 \text{ arcsec})(d/1 \text{ pc}) \text{ AU}, \quad P^2 a^3 = 4\pi^2 G M_{\text{BH}}$

$V_p(1-e) = V_a(1+e), \quad V_p^2 - V_a^2 = 2G M_{\text{BH}} a^{-1} [(1-e)^{-1} - (1+e)^{-1}]$
Distances From Motions

Example: Distance to supernova ring, if ring is circular
\[ t_{-} = \frac{R(1 - \sin i)}{c}, \quad t_{+} = \frac{R(1 + \sin i)}{c} \]

Fig 2.7 'Galaxies in the Universe' Sparke/Gallagher CUP 2007
Spectroscopic and Photometric Parallax

Assume stars of same spectral type and $Z$ have equal $L$.
Correct for dust reddening.
Works to 10% for main sequence stars,
60% for K giants.
Needs lots of observation time.

Cheaper alternative is to estimate spectral type from color, other indicators determine dwarf/giant.
Works best for clusters, where reddening is easier to determine.

Photometric distances towards South Galactic pole

$V - I$ colors of 12,500 stars with $m_V < 19$

$n_Z \propto e^{-|z|/h_z}$

5 < $M_V$ < 6
G, K stars

halo→
Mass-to-Light Ratio of Galactic Disc

$h_{R,S}$ is scale length of object of type $S$, $h_{z,S}$ is scale height.

$$n_S(R, z) = n_S(0, 0)e^{-R/h_{R,S}}e^{-|z|/h_{z,S}}$$

$$\Sigma_S(R) = 2n_S(0, 0)h_{z,S}e^{-R/h_{R,S}}, \quad I_S(R) = L_S\Sigma_S(R)$$

$$L_{D,S} = 2\pi I_S(0)h_{R,S}^2$$ is disc luminosity

Milky Way: $L_D \simeq 1.5 \times 10^{10} \, L_\odot$ in $V$ band, $h_R \simeq 4$ kpc

$I_D(4 \, \text{kpc}) \simeq 20 \, L_\odot \, \text{pc}^{-2}$, \quad $\sigma_D \simeq 40 - 60 \, M_\odot \, \text{pc}^{-2}$

$M/L_V = \sigma_D/I_D \simeq 2 - 3$ in disc; but about 1 for MS stars near the Sun

$M/L_V \sim 0.74$ for all stars except white dwarfs and gas

$M/L_V \sim 5$ from Galactic rotation
Analyzing Nearby Stars

- **Star Formation Rate**
  - $\mathcal{M}/L \sim 2$, $L_D \sim 1.5 \times 10^{10} \, L_\odot \implies \mathcal{M} \sim 3 \times 10^{10} \, M_\odot$.
  - $\tau_{gal} \sim 10 \, \text{Gyr}$, so if half of gas is returned to stars, star formation rate is $3 - 6 \, M_\odot/\text{yr}$.
  - Gas mass is $5 - 10 \times 10^9 \, M_\odot$, enough for 1–3 Gyr.

- $\mathcal{V}/\mathcal{V}_{\text{max}}$ test – a measure of the uniformity of a spatial distribution
  - An average of the ratio of the volume $\mathcal{V}$ from which an object could be seen divided by the volume $\mathcal{V}_{\text{max}}$ defined by the maximum distance at which an object in the sample is detectable.
  - Assume sample is of equally luminous objects and limited by a brightness limit which corresponds to the distance $z_{\text{max}}$ and the volume $\mathcal{V}_{\text{max}}$. A brighter object with $m < m_{\text{max}}$ originates from a smaller distance $z < z_{\text{max}}$.
  - $\frac{\mathcal{V}}{\mathcal{V}_{\text{max}}} = \frac{z^3}{z_{\text{max}}^3} \implies \left\langle \frac{\mathcal{V}}{\mathcal{V}_{\text{max}}} \right\rangle = \frac{\int_0^{z_{\text{max}}} z^5 \, dz}{z_{\text{max}}^3 \int_0^{z_{\text{max}}} z^2 \, dz}$
  - When $n(z)$ is uniform, $\left\langle \mathcal{V}/\mathcal{V}_{\text{max}} \right\rangle = 1/2$
Stellar Velocities and Scale Heights

Stars show the cumulative effects of acquiring random gravitational pulls from the lumpy disk: molecular clouds and dense star clusters. This translates into larger scale heights for older objects. Note that the Sun must be moving upwards at $7 \text{ km s}^{-1}$.

- $Z > 0.25Z_{\odot}$
- $Z < 0.25Z_{\odot}$

Nearby F and G stars
### Table 2.1 Scale heights and velocities of gas and stars in the disk and halo

<table>
<thead>
<tr>
<th>Galactic component</th>
<th>$h_z$ or shape</th>
<th>$\sigma_z = \sigma_R$ (km s$^{-1}$)</th>
<th>$\sigma_y = \sigma_\phi$ (km s$^{-1}$)</th>
<th>$\sigma_z$ (km s$^{-1}$)</th>
<th>$\langle v_y \rangle$ (km s$^{-1}$)</th>
<th>Fraction of local stars</th>
</tr>
</thead>
<tbody>
<tr>
<td>HI gas near the Sun</td>
<td>130 pc</td>
<td>$\approx 5$</td>
<td>$\approx 7$</td>
<td>Tiny</td>
<td>90%</td>
<td></td>
</tr>
<tr>
<td>Local CO, H$_2$ gas</td>
<td>65 pc</td>
<td>4</td>
<td></td>
<td>Tiny</td>
<td>90%</td>
<td></td>
</tr>
<tr>
<td>Thin disk: $Z &gt; Z_\odot/4$</td>
<td>(Figure 2.9)</td>
<td>$\approx 280$ pc</td>
<td>$\approx 32$</td>
<td>$\approx 42$</td>
<td>$\approx 45$</td>
<td>$\approx 10$</td>
</tr>
<tr>
<td>$\tau &lt; 3$ Gyr</td>
<td>$\approx 280$ pc</td>
<td>$\approx 27$</td>
<td>$\approx 32$</td>
<td>$\approx 42$</td>
<td>$\approx 45$</td>
<td>$\approx 10$</td>
</tr>
<tr>
<td>$3 &lt; \tau &lt; 6$ Gyr</td>
<td>$\approx 300$ pc</td>
<td>$\approx 32$</td>
<td>$\approx 33$</td>
<td>$\approx 42$</td>
<td>$\approx 45$</td>
<td>$\approx 10$</td>
</tr>
<tr>
<td>$6 &lt; \tau &lt; 10$ Gyr</td>
<td>$\approx 350$ pc</td>
<td>$\approx 42$</td>
<td>$\approx 45$</td>
<td>$\approx 50$</td>
<td>$\approx 55$</td>
<td>$\approx 10$</td>
</tr>
<tr>
<td>$\tau &gt; 10$ Gyr</td>
<td>$\approx 45$ pc</td>
<td>$\approx 50$</td>
<td>$\approx 55$</td>
<td>$\approx 60$</td>
<td>$\approx 65$</td>
<td>$\approx 10$</td>
</tr>
<tr>
<td>Thick disk</td>
<td>0.75–1 kpc</td>
<td>90%</td>
<td>$\approx 10$</td>
<td>$\approx 10$</td>
<td>$\approx 10$</td>
<td>$\approx 10$</td>
</tr>
<tr>
<td>$\tau &gt; 7$ Gyr, $Z &lt; Z_\odot/4$</td>
<td>(Figure 2.9)</td>
<td>68</td>
<td>$\approx 40$</td>
<td>$\approx 32$</td>
<td>$\approx 45$</td>
<td>$\approx 10$</td>
</tr>
<tr>
<td>$0.2 \lesssim Z/Z_\odot \lesssim 0.6$</td>
<td>63</td>
<td>39</td>
<td>39</td>
<td>$\approx 51$</td>
<td>$\approx 51$</td>
<td>$\approx 10$</td>
</tr>
<tr>
<td>Halo stars near Sun</td>
<td>$b/a \approx 0.5–0.8$</td>
<td>140</td>
<td>105</td>
<td>95</td>
<td>$\approx 190$</td>
<td>$\approx 10$</td>
</tr>
<tr>
<td>$Z \approx Z_\odot/50$</td>
<td>140</td>
<td>105</td>
<td>95</td>
<td>$\approx 190$</td>
<td>$\approx 10$</td>
<td>$\approx 10$</td>
</tr>
<tr>
<td>Halo at $R \sim 25$ kpc</td>
<td>Round</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>$\approx 215$</td>
<td>$\approx 10$</td>
</tr>
</tbody>
</table>

*Note: gas velocities are measured looking up out of the disk ($\sigma_z$ of HI), or at the tangent point ($\sigma_\phi$ for HI and CO); velocities for thin-disk stars refer to Figure 2.9. For thick disk and halo, abundance $Z$, shape, and velocities refer to particular samples of stars. Velocity $\langle v_y \rangle$ is in the direction of Galactic rotation, relative to the local standard of rest, a circular orbit at the Sun’s radius $R_\odot$, assuming $v_{y,\odot} = 5.2$ km s$^{-1}$.\*
Clusters and Associations

Disk stars are born from clouds large enough to gravitationally collapse.

- **Gould’s Belt (a Moving Group)**
  A ring centered 200 pc away containing the Sun and stars younger than 30 Myr. They lie in a layer tilted by $20^\circ$ about a line along the Sun’s orbit at $\ell = 90^\circ$.

- **Open clusters**
  $100 \, L_\odot < L < 3 \times 10^4 \, L_\odot$
  $\mathcal{M}/L < 1 \mathcal{M}_\odot/L_\odot$
  $0.5 \, pc < r_c < 5 \, pc$
  $\sigma_s < 0.5 \, km \, s^{-1}$

- **Globular clusters**
  $10^4 \, L_\odot < L < 10^6 \, L_\odot$
  $1 \mathcal{M}_\odot/L_\odot < \mathcal{M}/L < 2.5 \mathcal{M}_\odot/L_\odot$
  $0.5 \, pc < r_c < 4 \, pc$
  $25 \, pc < r_t < 85 \, pc$
  $4 \, km \, s^{-1} < \sigma_s < 20 \, km \, s^{-1}$
Distances and Ages of Clusters

100 Myr isochrone
100 Myr, w/o dust reddening
16 Myr isochrone

- Pleiades
  - blue giants, $\approx 5 \, M_\odot$
  - $(m - M)_0 = 5.6$

- binaries
- pre-main sequence stars

Fig 2.12 (J.-C. Mermilliod) 'Galaxies in the Universe' Sparke/Gallagher CUP 2007
Globular Clusters

47 Tuc

M92

[Fe/H] = −0.71
12 Gyr

Horizontal Branch

[Fe/H] = −2.15
13 Gyr

SMC red giants

12 ± 2 Gyr

13 ± 2 Gyr
Globular Cluster Distribution

Metal-rich clusters share Galactic rotation like stars in the thick disk.

Metal-poor clusters have highly eccentric, random orbits and do not share the Galactic disk rotation.

Fig 2.15 (D. Mackey) 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

Metal-poor [Fe/H] < -0.8  
Metal-rich [Fe/H] ≥ 0.8

♦ disrupted dwarf galaxy remnants

J.M. Lattimer  AST 346, Galaxies, Part 2
Halo Properties

- Oldest stars in halo a result of Galactic cannibalism.
  - Gravity slows galaxies during encounters and leads to mergers.
  - The Sagittarius dwarf galaxy is the closest satellite galaxy, and it is partially digested.
  - $\omega$ Cen is a bizarre globular cluster with non-uniform metallicity; possibly the remnant core of a digested satellite galaxy.
  - Magellanic Clouds will share the fate of Sagittarius dwarf galaxy in 3–5 Gyr.
  - Some believe the blue horizontal branch globular clusters joined the Milky Way after their galaxies were cannibalized.

- Metal-poor halo consists of globulars and metal-poor stars down to $[\text{Fe/H}] = 10^{-5}$
  - Most globulars have dissolved, forming many halo stars.
  - Palomer 5 has lost 90% of its stars and has tidal tails extending 10°.
  - Metal-poor halo stars form moving groups, including some carbon stars and M giants stripped from Sagittarius dwarf galaxy.
  - Total mass of halo metal-poor stars is about $10^7 \, M_\odot$.
  - Halo is somewhat flattened but rounder than bulge.
Sagittarius Dwarf Galaxy

J.M. Lattimer

AST 346, Galaxies, Part 2
The Halo as an Ensemble of Disrupted Satellites
The Bulge and Nucleus

- Bulge best studied with infrared light due to dust (31 mag extinction).
- Bulge accounts for 20% of Galaxy’s light with a size of order 1 kpc.
- Bulge is not dense inner portion of halo: stars are old but have average metallicity $> 0.5 Z_\odot$, $Z_{\text{max}} \sim 3 Z_\odot$.
- Average rotation speed 100 km s$^{-1}$ (half disk), stars have large random motions.
- Close to the center are dense, active star forming regions.
- Center has a 2 pc radius torus with $10^6 M_\odot$ of gas surrounding a $3 \times 10^7 M_\odot$ star cluster within 10″ (0.2 pc) including $> 30$ massive stars.
- Innermost stars $< 0.05$ pc from central black hole, $4 \times 10^6 M_\odot$ (Sag A*).
Sag A*

\[ R_0 = 8.28 \pm 0.1 \pm 0.29 \text{ kpc} \]

\[ M_{\text{BH}} = 4.30 \pm 0.20 \pm 0.30 \times 10^6 M_\odot \]
Differential rotation in Galaxy noticed by 1900 and explained by Oort in 1927.
Geometric Relations

\[
V_0 = V(R_0)
\]
\[
R_0 \sin \ell = R \sin(90^\circ + \alpha) = R \cos \alpha
\]
\[
R_0 \cos \ell = R \sin \alpha + d
\]
\[
\phi + \ell + \alpha = 90^\circ
\]
\[
V_r(R, \ell) = V(R) \cos \alpha - V_0 \sin \ell
\]
\[
= R_0 \sin \ell \left( \frac{V(R)}{R} - \frac{V_0}{R_0} \right)
\]
\[
V_t(R, \ell) = V(R) \sin \alpha - V_0 \cos \ell
\]
\[
= R_0 \cos \ell \left( \frac{V(R)}{R} - \frac{V_0}{R_0} \right) - d \frac{V(R)}{R}.
\]

When \(d << R\),

\[
V_r \approx R_0 \sin \ell \left( \frac{V(R)}{R} \right)'_{R_0} (R - R_0) = d \ A \sin(2\ell)
\]
\[
V_t \approx R_0 \cos \ell \left( \frac{V(R)}{R} \right)'_{R_0} (R - R_0) - d \frac{V(R)}{R}
\]
\[
= d[A \cos(2\ell) + B]
\]

\[
A \equiv -\frac{R}{2} \left( \frac{V(R)}{R} \right)'_{R_0}
\]
\[
B \equiv -\frac{1}{2R} [RV(R)]'_{R_0}
\]
\[
A + B = -V(R)'_{R_0}
\]
\[
A - B = V_0 / R_0
\]
\[ V(R) = V_0, \] equally luminous clouds in rings of radii 2, 3 \ldots 14 km spaced every degree in \( \phi \). Dot sizes indicate relative brightnesses.
Galactic Rotation Curve

For inner Galaxy, \( R < R_0 \), employ tangent point method. \( V_r(d, \ell) \) has a maximum along the line-of-sight at the tangent point where \( \alpha = 0 \):

\[
R = R_0 \sin \ell, \quad V(R) = V_r + V_0 \sin \ell.
\]

Method fails for \( R < 0.2R_0 \) since gas follows oval orbits in Galactic bar. For outer Galaxy, must use spectroscopic or photometric parallax.
Galactic Rotation Curve

Method of Merrifield (1992) for outer rotation curve. Local hydrostatic equilibrium dictates that the thickness $h_z$ of the HI layer is a function of $R$ but not $\ell$. A ring with constant thickness has a variable angular size:

$$\theta_b = 2 \tan^{-1} \left( \frac{h_z}{2d} \right), \quad d = R_0 \cos \ell + \sqrt{R^2 - R_0^2 \sin^2 \ell}.$$  

For circular rotation, and assuming co-rotation, the radial velocity satisfies

$$RV_r = \sin \ell \cos b (R_0 V(R) - RV_0), \quad \frac{V_r}{\sin \ell \cos b} = \frac{R_0}{R} V(R) - V_0 \equiv W(R).$$

Take data with fixed value of $W(R)$ and obtain the variation in angular width $\theta_b(\ell)$. Determine best fits for $R/R_0$ and $h_z/R_0$. Thereby we find $v_r(R)$ and $h_z(R)$.

$-75 \text{ km/s} < W < -70 \text{ km/s}$
Galactic Rotation Curve

Merrifield, AJ 103, 1552 (1992)
Implications of Galactic Rotation

For a spherical system,

\[ M(R) = \frac{RV(R)^2}{G} = 7.5 \times 10^{10} \left[ \frac{RV(R)^2}{R_0 V_0^2} \right] M_\odot. \]

Most of this is dark matter, and no reason to believe it is not spherically distributed. If \( \rho \propto r^{-2} \), the mass is linearly proportional to \( r \), matching the constant velocity rotation profile. Gauss’ Law \( \nabla^2 \Phi = 4\pi G \rho \) implies

\[ \Phi(r) = 4\pi G r_0^2 \rho_0 \ln(r/r_0), \quad \rho = \rho_0 (r_0/r)^2, \]

known as the singular isothermal sphere. To match the rotation of the Milky Way, we should have \( \rho_0 r_0^2 \simeq 0.9 \times 10^9 M_\odot \text{ kpc}^{-1} \).

A more realistic solution is one without the cusp at \( r = 0 \):

\[ \rho(r) = \rho_0 \frac{r_0^2}{r^2 + a^2}, \quad M(r) = 4\pi \rho_0 r_0^2 [r - a \tan^{-1}(r/a)]. \]

\[ V(r) = \sqrt{\frac{G M(r)}{r}} = \sqrt{4\pi G \rho_0 r_0^2} \sqrt{1 - \frac{a}{r} \tan^{-1} \frac{r}{a}}. \]

The velocity varies linearly near \( r \to 0 \) and vanishes as observed.