Exam # 2, 10 May

Lots of partial credit given, so give at least a qualitative answer to each item. Some useful constants are given at the end of the exam.

1. In this problem, you will explore a closed box model for galactic chemical evolution in the disk. In this model, assume there is no inflow or outflow, and also that there is instantaneous recycling. Make the usual assumptions that lead to the definitions of $p$, the amount of metals made in a stellar generation, and $R$, the return fraction. For the Salpeter mass function, $R \simeq 0.3$, so assume this value. Assume a Schmidt Law for the dependence of the star formation rate $\Psi$ on the surface mass density of gas in the disk, $\Sigma_g$:

$$\Psi (t) = \nu \Sigma_g^2 (t).$$

Note the exponent of 2 instead of 1 that we used in the lectures.

Determine the dependence of metallicity and the gas fraction $f_g = \Sigma_g / (\Sigma_g + \Sigma_*)$ on time in this model and adjust the remaining parameters of the model, $p$ and $\nu$, to fit the observations that the metallicity when the Sun formed was $Z_\odot$ and the gas fraction today $t = T$ is $f_g(T) = 0.26$. Assume both quantities initially were zero, $Z(0) = f_g(0) = 0$. At what values of $t$ were $[\text{Fe/H}]=-3$ and $[\text{Fe/H}]=-1$?

Determine the fraction of stars with metallicities between $\ln Z$ and $\ln Z + d \ln Z$, $(d\Sigma_*/d \ln Z) d \ln Z$, and determine where $d\Sigma_*/d \ln Z$ is a maximum. Sketch the shape of

$$\frac{d\Sigma_*}{d \ln Z} d \ln Z = \frac{d\Sigma_*}{d [\text{Fe/H}]} d [\text{Fe/H}]$$

and describe what is meant by the G-dwarf problem.

2. The Milky Way and the Andromeda galaxies are now $(t = T = 12$ Gyr) approaching each other at a velocity of 120 km/s and are currently 770 kpc apart. Neglecting the masses of the other members of the Local Group, and assuming they are in an elliptical orbit around each other with $e = 1$, how are $r$ and $t$ related? Assume at time $t = 0$ that $r = 0$, i.e., they were coincident.

How long will it be until they will next be coincident (i.e., $r = 0$)? Also determine the total mass of the two galaxies.
3. How do the colors and luminosities of disk galaxies vary with type? How do the mass-to-light ratios of disk galaxies vary with their luminosities, and what does this mean for the variation of the gas fraction of these galaxies? How do the observed average velocities of stars within galaxies vary with their luminosities? In disk galaxies, what is this relation called? In elliptical galaxies, what is this relation called? Qualitatively justify this relation for disk galaxies, using the Virial theorem, assuming the mass-to-light ratios and central surface brightnesses don’t vary among galaxies, and neglecting dark matter. What, if anything, should make you uncomfortable about this result?

4. Draw a spider diagram, a polar plot showing contours of constant radial velocities observed from a disk galaxy as a function of $R$ and $\phi$, where $R$ is the distance from the center of the disk and $\phi$ is longitudinal angle. Assume a flat rotation curve and that the inclination angle is $45^\circ$.

In a galaxy with a gravitational potential like the Plummer potential, $\Phi_G(R) = -G\mathcal{M}/\sqrt{R^2 + a^2}$ where $a$ is constant and $\mathcal{M}$ is the total mass of the galaxy, some of the contours are closed loops. See the figure below. Can you explain why some contours are closed?
5. What is meant by the terms “boxy” and “disky” as pertaining to elliptical galaxies? How does the ratio of ordered to random velocities, $V/\sigma$, compare for these types? What does this fact mean for the comparative shapes of these types?

How do relative brightnesses of these types compare?

How do the metallicities of elliptical galaxies vary with their luminosities?

Globally, what generic model of galaxy formation seems to best explain the above phenomena? What are the major problems associated with this model (i.e., what are some specific observations that are not readily explained)?

In the overall cosmic baryon budget, where do most baryons seem to reside?

Useful quantities: $1 \text{ pc} = 3.09 \times 10^{18} \text{ cm}$, $1 \text{ AU} = 1.5 \times 10^{13} \text{ cm}$, $N_0 = 1/m_H = 6 \times 10^{23} \text{ g}^{-1}$, $1 \mu \text{m} = 10^{-4} \text{ cm}$, $M_{V,\odot} \simeq 5$, $G = 4.3 \times 10^{-3} \text{ km}^2 \text{ s}^{-2} \text{ pc} \mathcal{M}_\odot^{-1}$. 