Black Holes

- What is a black hole?
- Who first thought of the concept of a black hole?
- Black holes and general relativity
- The “no-hair” theorem
- The event horizon
- Is there a singularity?
- Rotating black holes and the ergosphere
- Formation of black holes
- Hawking radiation, entropy, and black hole evaporation
- Orbits around black holes
- Observing black holes

Wikipedia
What is a Black Hole?

Standard definition: A region of space from which nothing, not even light, can escape.

- Where does the escape velocity equal the speed of light?

\[ v_{\text{esc}} = \sqrt{\frac{2GM_{BH}}{R_{\text{Sch}}}} = c \]

This defines the Schwarzschild radius \( R_{\text{Sch}} \) to be

\[ R_{\text{Sch}} = \frac{2GM_{BH}}{c^2} \sim 3 \frac{M}{M_\odot} \text{ km} \]

- The event horizon marks the point of no return for any object.
- A black hole is black because it absorbs everything incident on the event horizon and reflects nothing.
- Black holes are hypothesized to form in three ways:
  - Gravitational collapse of a star
  - A high energy collision
  - Density fluctuations in the early universe
- In general relativity, the black hole’s mass is concentrated at the center in a singularity of infinite density. 

J.M. Lattimer
Black Holes Large and Small
The first reference is by a geologist and Anglican priest, John Michell, in a letter written to Henry Cavendish, of the Royal Society, in 1783.

He argued that a Sun with 500 times its radius and the same density would be so massive that it’s escape velocity would equal light speed.

He reasoned, from observations of radiation pressure, that light, like a mass, has inertia. If gravity affects light like its mass equivalent, light would not be able to escape and would return to the surface.

He proposed using a prism to measure the gravitational weakening of starlight due to the surface gravity of the star.

Michell, around 1783, designed the experiment now attributed to Cavendish which first accurately measured the force of gravity between masses. This resulted in the first accurate values for $G$ and $M\oplus$.

He invented the torsion balance for the experiment but didn’t live to put it to use. His device passed to Henry Cavendish who performed the experiment in 1797-8.
Michell’s Apparatus – The Torsion Balance

$$\kappa \theta = LF = LGmM/r^2$$

$$T = 2\pi \sqrt{\frac{l}{\kappa}} = 2\pi \sqrt{\frac{mL^2}{2\kappa}}$$

$$G = \frac{2\pi^2 Lr^2}{MT^2 \theta}$$

$$M_\odot = gR_\odot^2 / G$$

Cavendish 1798, Phil. Trans. Roy. Soc. Lon., 469

Harvard Lecture Demonstration
Michell also tried to measure the radiation pressure of light, but when he focused sunlight onto a compass needle, it melted.

Other astronomical contributions include a study of parallax, which he acknowledged was too small to be presently observable, but would be in the future. He discussed how measurement of a star’s distance and apparent magnitude could be used to determine a star’s true luminosity.

He also proposed the explanation for twinkling of starlight.

In geology, he proposed that earthquakes were experienced as seismic waves of elastic compression travelling through the Earth, and determined the epicenter of the 1755 Lisbon earthquake.

He also first suggested that tsunamis were caused by earthquakes.

He defined the Mesozoic stratigraphic layer in the UK.
The mathematician Pierre-Simon Laplace made similar arguments in 1796 in the first two editions of his book *Exposition du systéme du Monde*, although the discussion was removed from later editions.

Karl Schwarzschild used Einstein’s newly developed theory of general relativity, in 1915, to find a solution applicable for a point mass or outside a spherical mass. (A few months later, Johannes Droste, Lorentz’s student, independently derived it.)

The solution had a bad behavior (a singularity) at the so-called Schwarzschild radius, but it was only later (1939) that Oppenheimer, Tolman and Volkov interpreted this radius as the boundary of a bubble where time stopped. This led to the idea of “frozen stars”.

In 1958, David Finkelstein identified the Schwarzschild radius with the event horizon, a perfect unidirectional membrane: causal influence can cross it in only one direction.

He and Martin Kruskal extended the Schwarzshild solution into the interior of the event horizon (so it could be applied to infalling observers) by means of a coordinate transformation.
In 1963, Roy Kerr found an analytic solution for the spacetime for rotating black holes.

In 1965, Ezra Newman found an analytic solution for charged and/or rotating black holes. However, it is not possible for black holes to attain a significant charge except in a complete vacuum.

In 1967, John Wheeler coined the name black hole.

Israel, Carter and Robinson evolved the no-hair theorem, which states that a black hole is completely described by just the mass, spin rate and electric charge. These are the only properties of a black hole visible from the outside.

The charge and spin of a black hole are limited:

\[ Q^2 + \left( \frac{J}{M} \right)^2 \leq M^2 \]

Violations of this limit lead to naked singularities which are not cloaked by event horizons. The cosmic censorship hypothesis forbids this from happening; it is supported by numerical simulations.
The Event Horizon

The boundary in spacetime through which matter and light can only pass in one direction. Thus, information cannot be extracted from inside it.

General relativity predicts that spacetime is deformed such that particle paths are bent towards the mass. At the horizon, no paths lead away from the black hole.

Time slows down near a black hole, compared to a distant observer. This leads also to gravitational redshift, as anticipated by Michell.
The Singularity

The event horizon is not a true singularity, but the center is. A non-rotating hole has a point singularity; a rotating one has a torus singularity. All the mass is contained within the singularity, which has infinite density.

Within a non-rotating hole, observers are inexorably carried into the singularity. In the case of a charged or rotating hole, it is possible for an observer to avoid the singularity and to re-emerge through the horizon into another spacetime, or even into one’s own past. These possibilities can occur only if the black hole has perfect symmetry, which won’t occur when the observer falls in.

There is also a boundary, known as the photon sphere, where photons moving tangentially to it are trapped in a circular orbit. This lies outside the event horizon. For a non-rotating black hole, \( R_{ph} = \frac{3GM_{BH}}{c^2} \). The orbit is unstable; any perturbation will cause ejection or injection.

The appearance of singularities is usually perceived as a breakdown of general relativity. However, it occurs in a domain in which quantum mechanics ought to be important. A unified theory of gravity and quantum mechanics is not yet possible.
The Ergosphere

A rotating black hole has a region, outside the event horizon, where it is impossible to “sit still”. The ergosphere is an oblate spheroid.

Any rotating object in general relativity “drags” an observer (frame dragging), but inside the ergosphere you’d have to travel faster than $c$ to avoid it.

If radiation or matter collides within the ergosphere, a projectile could exit the ergosphere with more energy than it entered with. The other projectile loses energy and falls in the event horizon. This Penrose process permits energy extraction from a black hole, which slows it down and lowers its mass.

A related process, the Blandford-Znajek mechanism, operates in the presence of a large magnetic field. This process could be important in powering active galactic nuclei and some gamma ray bursts.
The Blandford-Znajek Mechanism

Black Hole Thermodynamics and Evaporation

Stephen Hawking showed in 1974 that black holes can emit thermal radiation, which is caused by quantum mechanical tunneling through the event horizon.

The radiation rate and spectrum is that of a perfect blackbody

\[ L = 4\pi R_{Sch}^2 \sigma T_{BH}^4 \]

where the black hole temperature is

\[ T_{BH} = \frac{\hbar c^3}{4Gk_B M_{BH}}. \]

Therefore, over time, black holes evaporate. However, black holes above a certain mass (3 × 10^{-8} M_\odot, i.e., the Moon’s mass) have \( T_{BH} < 2.7 \) K and gain more mass by swallowing the cosmic background radiation than they lose by evaporation; they will grow with time.

Since \( L \propto M_{BH}^{-2} \) the time-to-evaporation is \( \tau_{BH} \propto M_{BH}^3 \). For a 1M_\odot hole, \( T_{BH} \approx 10^{-7} \) K and \( \tau_{BH} \approx 10^{74} \) s.
Hawking showed in 1971 that the total area of the event horizons of merging black holes cannot decrease. This seems analogous to the *second law of thermodynamics*, which states that the total entropy of a system cannot decrease.

Bekenstein postulated that the area is equivalent to the entropy of a black hole:

\[ S_{BH} = \frac{k_B c^3}{4\hbar G} A_{BH} = \frac{4\pi G k_B}{\hbar c} M_{BH}^2 \]

Hawking's discovery is known as the *second law of black hole thermodynamics*.

There are analogous *zeroth* and *first laws of black hole thermodynamics* if mass acts as energy, area as entropy, and surface gravity as temperature:

\[ g_{BH} = \text{constant}, \quad dM_{BH} = \frac{g_{BH}}{8\pi} dA_{BH}; \quad T = \text{constant}, \quad dE = T dS \]

Gerard 't Hooft and Leonard Susskind proposed the *holographic principle*, anything happening within a volume can be described by data on its boundary, based on the connection between entropy and black hole area.
How Are Black Holes Created?

- Black holes are the end product of gravitational collapse.
- At the end of its life, a star has a degenerate core in which most of the pressure is not thermal but due to the Fermi Exclusion Principle: no two fermions can occupy the same momentum and spin state simultaneously.
- Electrons (fermions) pile on top of each other with greater and greater energies, producing pressure that resists gravity.
- Chandrasekhar showed that the upper mass limit for a degenerate electron core is about 1.4 $M_\odot$, the so-called Chandrasekhar Mass $M_{Ch}$.
- A star with less than 5 or so $M_\odot$ sheds its outer envelope and is left with a degenerate core of less than $M_{Ch}$, which cools into a stable white dwarf.
- Larger stars have more massive cores. Once nuclear burning ceases, these cores are unstable and collapse.
- Collapse halts due to the repulsive nuclear force: a proto-neutron star is born and a shock is produced that may ultimately eject the star’s envelope.
- But a neutron star also has a maximum stable mass, estimated to be between 2 and 3.2 $M_\odot$. Remnants with more mass will undergo a second collapse, forming a black hole.
Detection of Stellar Black Holes

Most stellar black holes are members of X-ray binaries in which matter is accreted from a donor star. Infalling matter onto the black hole releases gravitational potential energy (up to about 30% of its rest mass energy).

The maximum rate of spherical accretion is given by the Eddington limit. This is where radiation pressure balances gravity. The radiative flux from accretion falls as $1/r^2$ as does gravity, so the Eddington limit depends only on the source’s mass $M$ and the opacity $\kappa$ of the accreting matter:

$$L_{Edd} = \frac{4\pi cGM}{\kappa} = 1.3 \times 10^{38} \left( \frac{M}{M_\odot} \right) \text{erg s}^{-1}.$$

With the blackbody formula $L = 4\pi R^2 \sigma T^4$, the effective temperature of emission is

$$T_{\text{eff}} = \left( \frac{cGM}{\sigma \kappa R^2} \right)^{1/4} \approx 2 \times 10^7 \text{ K}.$$

This corresponds $h\nu = k_B T_{\text{eff}} \approx 1.6 \text{ keV X-rays}$. 

Wikipedia

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Black Holes Large and Small
Determination of Black Hole Masses in Binaries

Kepler’s 1–2–3 Law

\[
\frac{G(M_X + M_C)}{a^3} = \left(\frac{2\pi}{P_{orb}}\right)^2
\]

Companion velocity

\[
v_C = \frac{2\pi}{P_{orb}} a_C \sin i
\]

Center of mass

\[a M_X = (M_X + M_C) a_C\]

Combine these:

\[
\frac{v_C^3 P_{orb}}{2\pi G} = \frac{(M_X \sin i)^3}{(M_X + M_C)^2} = f
\]

Note that \(M_X > f_C\).

For Cyg X-1, \(i < 60^\circ\), \(P_{orb} = 5.6\) days, \(f = 0.244 \pm 0.005\) \(M_\odot\), and the mass of the B0 supergiant companion HDE 226868 is \(M_C > 20\) \(M_\odot\), leading to \(M_X > 7\) \(M_\odot\).

http://www.spacetelescope.org/extras/posters/cygnus_x1/
Confirmed Stellar Black Hole Masses

Casares (2006)
Intermediate Mass Black Holes

Intermediate mass black holes have masses $30 < \frac{M}{M_{\odot}} < 30,000$. To date, among the suspects:

- **GCIRS 13E**, which is orbiting 0.4 light-years from the supermassive black hole (Sag A*) at the Galaxy’s center. It has a mass of $1300 M_{\odot}$ and lies within a cluster of 7 other stars, a possible remnant of a globular cluster stripped by interactions with Sag A. Runaway stellar collisions might have produced this black hole.

- **M82 X-1**, about $500 M_{\odot}$, which is orbited by an evaporating red giant.

Both have been questioned.
In addition, it is suspected that many globular clusters might harbor intermediate mass black holes. They are detectable from larger-than-normal velocities of stars near the cluster centers.
Supermassive black holes have masses from $10^5 - 10^{10} \, M_\odot$. Possibly all galaxies, including the Milky Way, have supermassive black holes at their centers.

Compared to stellar holes, tidal forces in their vicinity are considerably weaker.

They are detectable due to the fast Keplerian motion of nearby gas, stars and water masers.

There is a strong correlation between the velocity dispersion of galaxies and their central hole masses. The tightness indicates a strong feedback between hole growth and bulge mass.

They are believed to be the "central engine" of active galactic nuclei, like Seyfert galaxies and quasars.
The Center of the Milky Way
Supernova Remnants Near the Galactic Center

http://cassfos02.ucsd.edu/public/tutorial/MW.html
300 Light-Years Around the Galactic Center

Hubble/Spitzer
Stars Near the Galactic Center

ESO/S. Gillessen et al.

Stellar orbits

The Centre of the Milky Way (VLT YEPUN + NACO)

ESO PR Photo 23a/02 (9 October 2002)

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Black Holes Large and Small

Stellar orbits

1 light-year

0.025 lt-yr
Fitting positions and radial velocities of star S2 yields a mass $(4.1 \pm 0.6) \times 10^6 \, M_\odot$. The error is primarily due to uncertainties in the distance to the Galactic center.
Two supermassive black holes \((3 \times 10^7 \, M_\odot \text{ and } 10^6 \, M_\odot)\) discovered orbiting at a separation of 490 light-years in the galaxy NGC 3393, about \(1.6 \times 10^8\) light-years distant. They will collide in about 10,000 years.
GRB110328A wasn’t a normal GRB. It has persisted for months and originates from a galactic core 3.8 billion light-years distant. No previous evidence of X-ray or UV emissions. The burst is a jet from the tidal disruption of a star and we are looking down the barrel.
A supermassive black hole of about $10^6 \ M_\odot$ has been inferred to exist in a blue compact dwarf galaxy Henize 2-10 which has a concentrated region of extreme star formation.

It is actively accreting mass and emitting X-rays.

Although this dwarf galaxy is nearby, it resembles galaxies thought to exist in the infant Universe which don’t have substantial spheroidal components.

Therefore, it appears supermassive black hole birth precedes the buildup of galaxy spheroids.
Can We Directly Detect an Event Horizon?

- Event horizon of Sgr A* has largest angular size of any black hole in the Universe, $10 \mu$ arc-sec.
- VLBI observations of Sgr A* show disc emission on a scale of $37 \mu$ arc sec, already evidence for the existence of an event horizon.
- Submillimeter VLBI observations might have enough time resolution to detect periodicity in emissions from hot spots at the innermost stable circular orbit, which will allow the determination of the black hole spin.

A proposed submillimeter VLBI “Event Horizon Telescope” will produce images of the Galactic center showing silhouettes predicted by general relativistic lensing.

- The same techniques are applicable to M87.

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Model of 345 GHz emission

Simulated image 7-telescope array

Simulated image 13-telescope array

A. Broderick

Fish & Doeleman (2010)

Fish & Doeleman (2010)
NASA - Astronomers have discovered a pair of supermassive black holes in the twisty, spiral brain of former Alaskan governor Sarah Palin, NASA said Wednesday.

Approximately 160 million light years from Earth, Ms. Palin is the nearest known such phenomenon, said scientists at NASA’s FarOut X-ray Observatory.

A supermassive black hole is the largest type of black hole in a galaxy, in the order of hundreds of thousands to billions of solar masses. It is known for sucking up all the matter, whether it be stars, planets, or even smaller black holes. However, NGC 3393 is a disorganized spiral brain, full of contradictions, cosmetics and creepy theories about tin foil hats, and its central bulge is dominated by old stars like Buddy Hackett and that fourth Marx Brother that no one can ever remember the name of. This makes the possibility of merger more remote.