The Sun: Example of Radiation Laws

- \( L_\odot = 4 \cdot 10^{33} \text{ erg/s} \)

- Its yellow color means the peak wavelength of the Sun’s spectrum is \( \lambda_{max} \approx 5 \cdot 10^{-5} \text{ cm} \)

- Wien’s Law establishes the Sun’s surface temperature: \( T_\odot = 0.29 \text{ cm}/\lambda_{max} \approx 5800 \text{ K} \)

- Invert the blackbody luminosity formula to estimate the solar radius:

\[
R_\odot = \sqrt{\frac{L_\odot}{4\pi\sigma T_\odot^4}} = 7 \cdot 10^{10} \text{ cm}
\]
General Properties of Stars

- Masses: from 0.1 \( \text{M}_\odot \) to 100 \( \text{M}_\odot \)
- Luminosities: from 0.0001 \( \text{L}_\odot \) to \( 10^6 \text{L}_\odot \)
- Radii: from 0.1 \( \text{R}_\odot \) to 1400 \( \text{R}_\odot \)
- Surface Temperatures: from 2000 K (mostly infrared radiation) to 40,000 K (mostly ultraviolet radiation).
- Some “stars”, white dwarfs and neutron stars, are much more compact and have extreme properties.
- Objects with masses between a few Jupiter masses and 0.1 \( \text{M}_\odot \) don’t burn nuclear fuel and are called \textbf{brown dwarfs}: they are neither planets nor stars.
Properties and Types of Stars

The main physical properties of stars are their luminosity $L$, surface temperature $T$, radius $R$ and mass $M$. E. Hertzsprung & H. Russel found that plotting $L$ vs. $T$ was a useful way to discriminate types of stars.
Most stars are Main Sequence (M-S) stars, which burn H into He in their cores.

Other groups are red giants, which have exhausted H fuel and “burn” He into C and O, supergiants which are burning even heavier elements, and white dwarfs (dead low-mass stars).

Main Sequence stars can be divided into spectral types O, B, A, F, G, K and M based on temperature or color.

Note that there are diagonal lines of constant radius.
## Spectral Types

<table>
<thead>
<tr>
<th>Spectral Type</th>
<th>Initial Mass (M☉)</th>
<th>Initial Lum. (L☉)</th>
<th>$T_{surf}$ °K</th>
<th>Life (years)</th>
<th>Radius (R☉)</th>
<th># in Galaxy</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>60</td>
<td>800,000</td>
<td>50,000</td>
<td>$1 \cdot 10^6$</td>
<td>12</td>
<td>$5.5 \cdot 10^4$</td>
</tr>
<tr>
<td>B</td>
<td>6</td>
<td>800</td>
<td>15,000</td>
<td>$1 \cdot 10^8$</td>
<td>3.9</td>
<td>$3.6 \cdot 10^8$</td>
</tr>
<tr>
<td>A</td>
<td>2</td>
<td>14</td>
<td>8000</td>
<td>$2 \cdot 10^9$</td>
<td>1.7</td>
<td>$2.4 \cdot 10^9$</td>
</tr>
<tr>
<td>F</td>
<td>1.3</td>
<td>3.2</td>
<td>6500</td>
<td>$6 \cdot 10^9$</td>
<td>1.3</td>
<td>$1.2 \cdot 10^{10}$</td>
</tr>
<tr>
<td>G</td>
<td>0.9</td>
<td>0.8</td>
<td>5500</td>
<td>$1.3 \cdot 10^{10}$</td>
<td>0.92</td>
<td>$2.8 \cdot 10^{10}$</td>
</tr>
<tr>
<td>K</td>
<td>0.7</td>
<td>0.2</td>
<td>4000</td>
<td>$4 \cdot 10^{10}$</td>
<td>0.72</td>
<td>$6 \cdot 10^{10}$</td>
</tr>
<tr>
<td>M</td>
<td>0.2</td>
<td>0.01</td>
<td>3000</td>
<td>$2.5 \cdot 10^{11}$</td>
<td>0.3</td>
<td>$3 \cdot 10^{11}$</td>
</tr>
</tbody>
</table>
M-S Properties Related

• A star’s physical properties on the Main Sequence (M-S) are related: \( T_{\text{center}} \propto M/R; \quad R \propto M^{1/2} \)

• A star radiates like a blackbody, so \( L \propto R^2T^4 \)

• Approximately \( T_{\text{center}} \propto T_{\text{surface}} = T \).

• Combining the above, \( L \propto M^3 \)

• A star’s Main Sequence lifetime can be estimated by considering the amount of fuel and dividing by the rate at which the fuel is burned:
  \( \tau \propto M/L \propto M/M^3 = M^{-2}; \quad \tau = M/L\tau_\odot \)

• Massive stars burn out too quickly for life to form.

• But low-mass stars have a smaller habitable zone, whose volume \( V \propto M^4 \propto L^{4/3} \).
Stellar Habitable Zones
Advanced Evolution of (Low-Mass) Stars

- Low-mass stars ($M < 8M_\odot$) evolve into a red giant phase in which their surfaces expand enormously and also cool, but their interiors shrink and heat. This occurs when the H fuel in the star’s core is depleted. A red giant burns He into C and O. When the He fuel is eventually exhausted, the outer stellar portions expand away (planetary nebula) and core cools and dies as a white dwarf.
White Dwarfs

• Normal stars are composed of gas supported by thermal pressure, where \( P \propto \rho T \).
• Ordinary solid matter at room temperature, like rock, has essentially zero pressure.
• As the temperature is lowered, gas pressure tends to zero. So when a star dies and subsequently cools off, little pressure exists to support it against gravity even if it crystallizes (freezes).
• However, quantum mechanics predicts, because of the Pauli Exclusion Principle, that it is difficult to squeeze electrons or nucleons into too small a volume. This additional source of pressure, which exists even at zero temperature, is called degeneracy pressure.
• A star as it cools off eventually replaces thermal pressure with degeneracy pressure and becomes a white dwarf.
• Degeneracy pressure is large, but not infinite. Chandrasekhar showed that an upper limit to the white dwarf mass is about 1.4 M\(_\odot\) (called the Chandrasekhar limit). This is the limit obtained from electron degeneracy pressure.
• Typical white dwarfs have masses within 0.1 – 1.2 M\(_\odot\), and radii 3 R\(_\oplus\) \(\simeq\) 0.01 R\(_\odot\).
• The density in a white dwarf is about \(10^6\) g cm\(^{-3}\), a million times that of water (a teaspoon of a white dwarf would weigh as much as an elephant on the Earth).
Evolution in the H-R Diagram

[Diagram showing the Hertzsprung-Russell diagram with labels for various stages of stellar evolution, including the Sun in its current state as an A-type star, moving towards the red giant branch, and eventually transitioning into a white dwarf.]
Proof of Evolution

M55

Cluster Main Sequence

WD → Blue Stragglers

Horizontal Branch

Red Giant Branch

Asymptotic Giant Branch

Blue Dwarf Stars

White Dwarf

Population Red Dwarf Stars

Stars Evolving to Red Giant Stage

Background Stars

Blue Dwarf Stars

M4

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[Graph showing the H-R diagram with M55 and M4 clusters, indicating various stellar phases and evolutionary stages]
High Mass Stars Make Type II Supernovae

- High-mass stars ($M > 8M_\odot$) evolve into both red and blue giants and supergiants as their cores burn heavier and heavier elements.

- After the core is converted into iron, the most bound element, nuclear burning ceases to produce energy and the star collapses to form a neutron star or black hole, a violent event accompanied by a supernova explosion (of Type II).

- Supernovae within a few thousand lt.-yrs. would be lethal to Earth life.

- Yet life could not exist without supernovae, because supernovae eject nearly all heavy elements into space. Until supernovae occurred, gas from which stars and planets were made contained virtually no elements heavier than helium.
Stellar Habitable Zones

Wikipedia

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Advanced Evolution in Binary Systems

- Stars in binaries may evolve and die differently than single stars.
- Stars in close binaries may transfer mass, for example, when one becomes a red giant or supergiant.
- White dwarfs in close binaries may accrete matter transferred from a stellar companion. White dwarfs have an upper mass limit \( M_{\text{Chandrasekhar}} \approx 1.4 M_\odot \).
- Accreting sufficient mass, they may begin to collapse. However, they consist of nuclear fuel (e.g., C, O) and instead explode like a super thermonuclear bomb. The C and O is incinerated into iron in a violent Type I supernova.
**Nuclear Nomenclature**

- An atom is composed of a nucleus and 1 or more electrons.
- The nucleus has $1/100,000$ the radius of the atom, but nearly all its mass.
- Nuclei are composed of positively charged protons and neutral neutrons.
- Elements are distinguished by the atomic number $Z$ (number of protons).
- $Z$ (element symbol)$^A$ refers to a nucleus or an atom of atomic number $Z$ and atomic weight $A = Z + N$.
- An isotope is a nucleus with the same $Z$ as another nucleus but with a different neutron number $N$. Chemically, isotopes are nearly identical.
- A nucleus that is not permanently stable is radioactive and eventually decays into another. Although the decay of a particular radioactive nucleus is random, 50% of a collection of radioactive nuclei decays in one half-life.
Periodic Table of Rejected Elements

Bosphorus

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The Atlantic Monthly, Aug. 1999; The Periodic Table of Rejected Elements - 99.08; Volume 284, No. 2; page 43.
Nuclear Energy Generation

• In general, fusion in stars goes from the lightest elements to heavier ones, because Coulomb repulsion is proportional to $Z_1 Z_2$.

• Fusion of hydrogen nuclei to helium nuclei (proton-proton cycle)

\[
\begin{align*}
1\text{H}^1 + 1\text{H}^1 & \rightarrow 1\text{H}^2 + e^+ + \nu_e; \\
1\text{H}^2 + 1\text{H}^1 & \rightarrow 2\text{He}^3 + \gamma; \\
2\text{He}^3 + 2\text{He}^3 & \rightarrow 2\text{He}^4 + 1\text{H}^1 + 1\text{H}^1.
\end{align*}
\]

$\nu_e$ is the neutrino, a massless nearly invisible particle.

• R. Davis (Brookhaven Nat'l Lab) first detected neutrinos from the Sun using a tank of 100,000 gallons of carbon tetrachloride ($\text{C}_2\text{Cl}_4$, cleaning fluid) in the Homestake Gold Mine in Lead, South Dakota in the 1970's. The observation of neutrinos from the Sun proves that energy is generated by the fusion of H into He: every time a proton is converted into a neutron, a neutrino is produced.

• In the red giant phase, helium is converted into heavier elements by

\[
\begin{align*}
3(2\text{He}^4) & \rightarrow 6\text{C}^{12} \\
6\text{C}^{12} + 2\text{He}^4 & \rightarrow 8\text{O}^{16}
\end{align*}
\]

• There are no stable elements of mass 5 or 8. This is why the Big Bang doesn’t produce many nuclei heavier than helium, and why a rare three-body collision is needed to synthesize heavy elements in stars.
In massive stars, nuclear burning of heavier elements continues: C, O $\rightarrow$ Ne $\rightarrow$ Mg, Si $\rightarrow$ S, Ca $\rightarrow$ Fe, Ni

Beyond C, O burning, more energy is wasted in producing neutrinos than in delivering heat to support the star against gravity and in generating its emitted light. The neutrinos escape rather than give up their energy inside the star.

The interaction of neutrinos with matter is so weak that to have a 50% chance of stopping a solar neutrino, a lead shield would have to be 100 lt.-yrs. thick.

The production of neutrinos reaches a crescendo in the gravitational collapse leading to a Type II supernova. The luminosity of neutrinos for a few seconds then rivals the energy output of all the stars in all the galaxies of the visible universe! The energy emitted in neutrinos within a few seconds in a supernova is more than 300 times the total light output of a solar-like star during its entire life!

The production of neutrinos and the nucleosynthesis and ejection of heavy nuclei in Type II supernovae was confirmed by SN 1987a in the Large Magellanic Cloud, a nearby galaxy, on February 23, 1987. Neutrino detectors in Ohio and Japan detected a total of about 20 neutrinos even though this supernova was 180,000 lt.-yrs. from the Earth. Radiactive Ni, Co and Ti nuclei were also observed in the ejecta. To date, no neutron star is visible within the remnant, suggesting that a black hole might have formed instead.
Neutron Stars and Black Holes

- If matter is forced to even higher densities than in a white dwarf, \(10^6\) times that of water, it collapses but could stabilize to form a neutron star with aid of additional pressure from nucleon degeneracy and the strong nuclear force.

- Neutron stars have masses 1.2–1.5 \(M_\odot\) and radii 10–15 km, only 3–4 times the radius of the same mass black hole.

- The upper mass limit to a neutron star is not well known, but has to be less than about 3 \(M_\odot\).

- Without the existence of degeneracy pressure, a cold star would eventually collapse into a black hole, first predicted by Mitchell in 1783, but termed \textit{corps obscura} by Laplace in 1795.

- A black hole occurs when the size of the object becomes small enough that the escape velocity becomes equal to the speed of light \(c\).

\[
v_{\text{escape}} = \sqrt{\frac{2GM}{R}}, \quad \Rightarrow \quad R = 2\frac{G}{c^2} M \approx 3\frac{M}{M_\odot} \text{ km}
\]

- Cold objects of larger mass than the upper limit to the neutron star mass must be black holes. Many black holes are known to exist, with masses ranging from about 10 \(M_\odot\) to millions of solar masses.

- Masses of white dwarfs, neutron stars and black holes have been precisely measured in binary systems using Kepler’s Laws with general relativistic refinements.
Nucleosynthesis

- After the Big Bang, matter in the universe consisted almost entirely of only H and He.
- Heavy elements are synthesized only in stars.
- Heavy elements are mostly dispersed through supernova explosions, of both Type I and Type II.
- Some C, N, O and Ne are also ejected from stars in stellar winds and in novae (when “combustible” H is accreted, burned, and ejected from the surface of a white dwarf in a close binary).
- Because of Coulomb repulsion, the abundance of elements decreases with $Z$.
- Elements made of multiples of $\alpha$—particles (He nuclei) are more abundant than average.
- Elements with even numbers of protons are more abundant than odd $Z$ elements.
- Fe and Ni are the most bound elements, so heavier elements are created only in rare explosive events like supernovae.