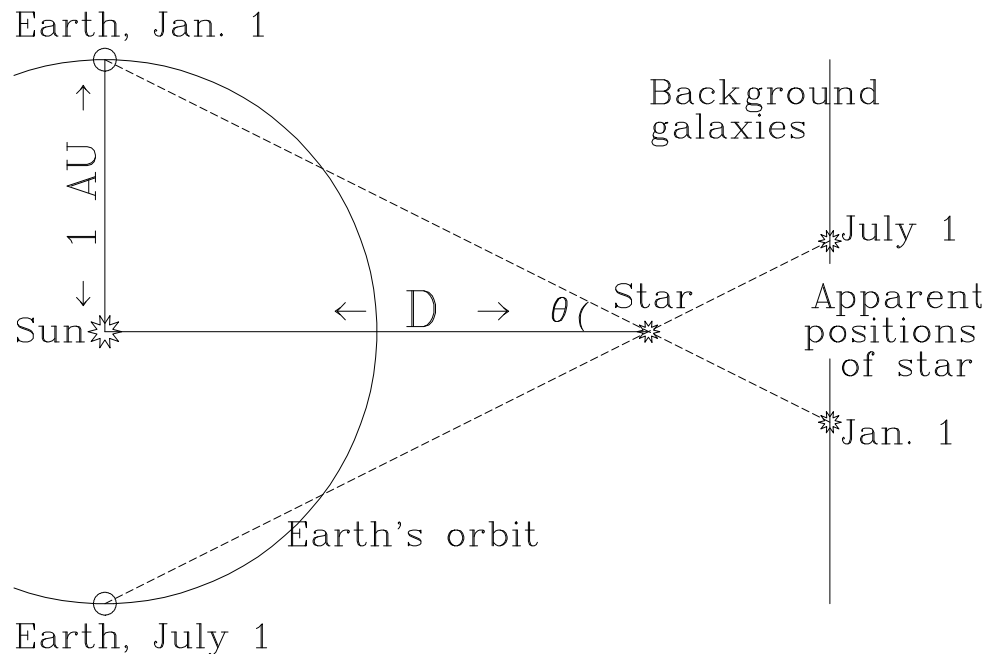


Distances in Astronomy

- Radius of the Earth (\oplus): $R_{\oplus} = 6.4 \cdot 10^8 \text{ cm} = 3977 \text{ miles}$
- Radius of the Sun (\odot): $R_{\odot} = 7 \cdot 10^{10} \text{ cm} = 435,000 \text{ miles}$
- Earth–Sun distance: 1 Astronomical Unit (1 AU) = $1.5 \cdot 10^{13} \text{ cm} = 93,000,000 \text{ miles}$
- 1 light-year = $9.5 \cdot 10^{17} \text{ cm}$
Note that 1 year = $3.16 \cdot 10^7 \text{ seconds}$ and $c = 3 \cdot 10^{10} \text{ cm/s}$,
so 1 lt.-yr. = $3.16 \times 3 \times 10^{7+10} \text{ cm}$
- 1 parsec (pc) = $1 \text{ AU}/\tan(1'')$ = $3.1 \cdot 10^{18} \text{ cm} = 3.26 \text{ lt.-yr.}$

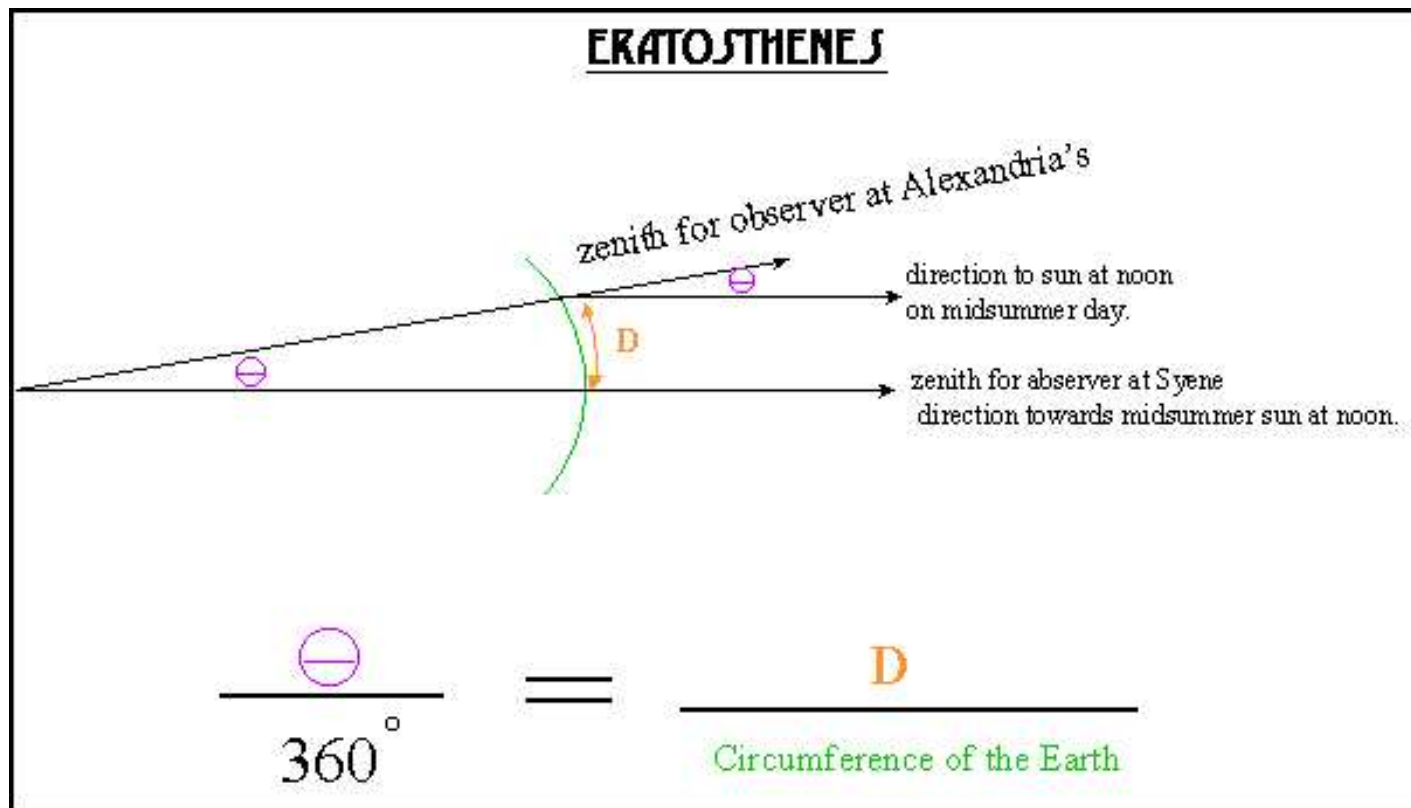


- $D(\text{pc}) = 1/\theta('')$ since $\tan \theta \simeq \theta$ for very small angles.
- 1 kpc = 10^3 pc ; 1 Mpc = 10^6 pc

Using Geometry to Measure Distances

- Size of the Earth: Eratosthenes (about 240 BC)

Eratosthenes knew that at noontime on the summer solstice (June 21) the Sun was directly overhead at Syene (southern Egypt) because the bottom of a vertical well was illuminated. At the same time in Alexandria, which is due north, the Sun cast a shadow which Eratosthenes measured to be at an angle of $\theta = 7^\circ$ to the vertical, about 1/50 of a complete circle. The estimated distance between the two cities was about $D = 500$ miles. The circumference of the Earth must be 50 times 500 miles or 25,000 miles.



Using Geometry to Measure Distances

- Earth–Moon: Aristarchus (270 BC)

In a total lunar eclipse, the Moon passes through the Earth's shadow. Aristarchus observed that it took about 3 hours for the mid-point of the Moon to cross the center of the shadow. He reasoned that the width of the Earth's shadow is the same as its diameter. If the Moon moves around the Earth at a constant speed, and it takes about 29.5 days for the Moon to complete one orbit, then

$$\frac{\text{Circumference of orbit}}{\text{Diameter of Earth}} = \frac{\text{Lunar month}}{\text{Duration of eclipse}} = \frac{2\pi R}{2R_{\oplus}} = \frac{29.5 * 24}{3} = 236$$

Then the distance to the Moon is $236/\pi = 75$ times the Earth's radius. The correct value is 60. The Earth's shadow is a cone, and is 25% smaller at the Moon.

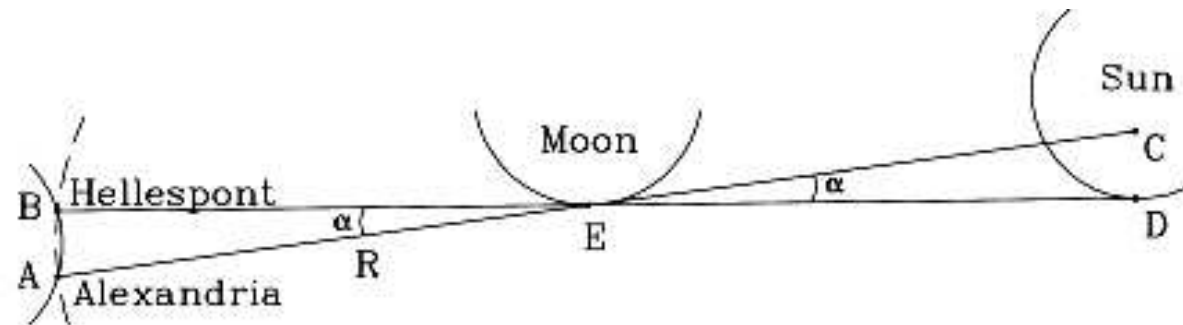
- Earth–Moon: Hipparchus (128 BC)

In a total solar eclipse, the Moon barely covers the Sun. Hipparchus used the fact that while the eclipse was total at Hellespont (Turkey) it was only 4/5 at Alexandria. The latitudes of these two cities differ by 9° and are north-south of each other. Also, since the angular diameter of the Sun (and Moon) is 0.5° , the angle $\alpha = 0.1^{\circ}$ since the eclipse was 4/5 total at Alexandria.

$$AB = 2\pi R_{\oplus} \frac{9}{360} = 2\pi R_{\oplus} \frac{0.1}{360}$$

Thus the distance to the Moon is $R = 9R_{\oplus}/.1 = 90R_{\oplus}$. The error is because the Moon was not

overhead at the time of the ecipse, but at an angle of $40\text{-}45^{\circ}$ from the zenith.



Using Geometry to Measure Distances

- Distance to the Sun: Aristarchus (about 270 BC)

Aristarchus felt that the Moon orbits the Earth and the Earth-Moon system orbits the Sun. He reasoned that when the Moon was exactly half lit by the Sun that the Earth-Moon-Sun angle was 90° . If the Sun was infinitely far away, then the angle α would also have to be 90° and the angle $\beta = 0^\circ$.

Aristarchus estimated however, that the angle β

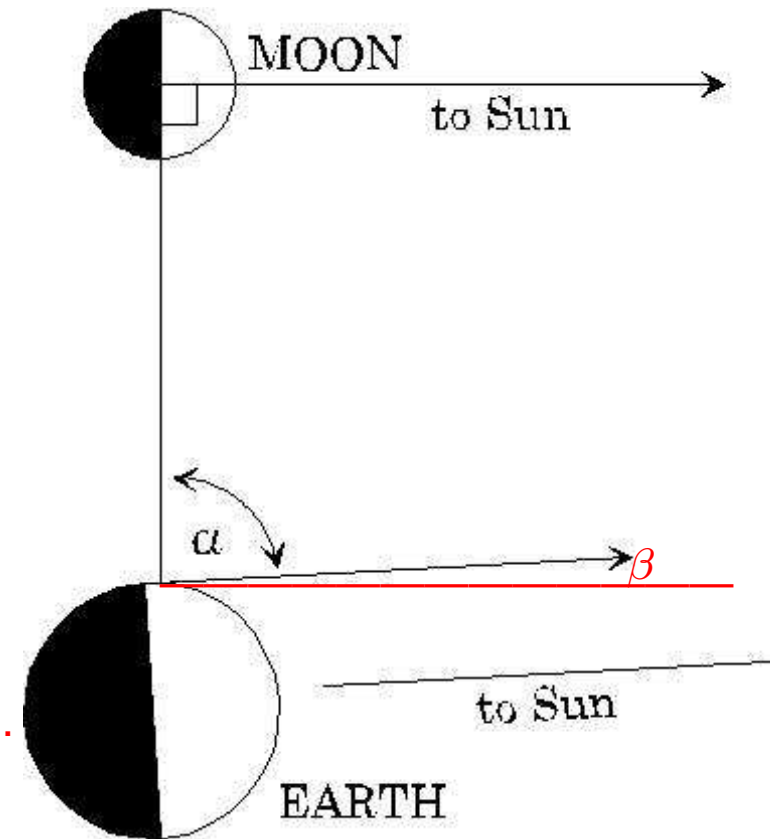
was about 3° , and $\beta = \frac{D_{M-E}}{D_{E-S}} \frac{360}{2\pi}$.

Therefore $D_{E-S}/D_{E-M} = \frac{360}{2\pi\beta} = 19$.

This is too small by about a factor of 20.

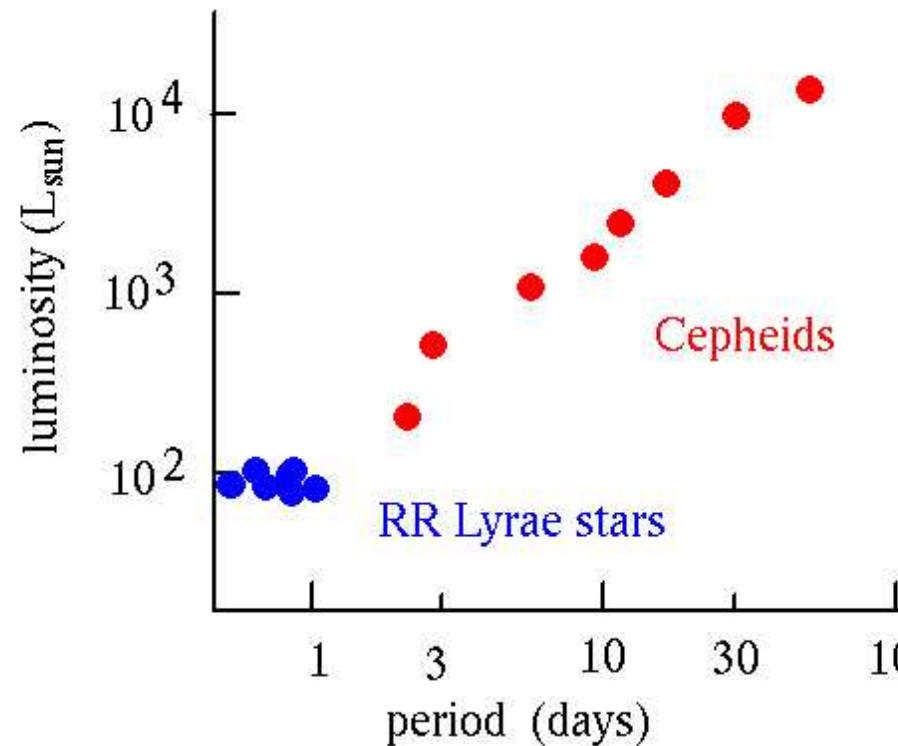
- Distance to the Sun: Cassini and Richer (1672)

They used a measurement of the parallax of Mars to determine the distance to Mars, and then inferred the distance to the Sun using Kepler's Laws. Richer traveled to Cayenne, French Guiana, while Cassini remained in Paris to simultaneously measure the location of Mars in the sky at a relatively near approach. They obtained a value 20 times larger than Aristarchus' which had been the accepted value up to that point.



Cosmic Distance Ladder

- **Parallax** Establish a database of stellar distances to thousands of nearby stars.
- **Inverse Square Law of Brightness** Stars established to be similar (color, spectra, etc.) have similar luminosities. Their relative brightnesses then yield their relative distances.
- **Statistical, Secular and Moving Cluster Parallaxes**
- **Cepheid and RR Lyrae Variable Stars as Standard Candles** Henrietta Leavitt discovered in 1912 the period-luminosity relation among Cepheid variables. They are now used as *standard candles*.
- **Velocity Measurements With Redshifts**
Planetary nebulae, novae, supernovae, orbiting stars.
- **Tully-Fisher Relation** Relation between the observed velocity width (amplitude of rotation curve) and luminosity of spiral galaxies.
- **Type I Supernovae** Exploding white dwarfs of similar masses yield similar luminosities and therefore are good standard candles.
- **Redshifts of Galaxies and Quasars**



The Universe Reduced by a Billion

| Object | Size | Distance from Earth |
|-------------------|------------------------|-----------------------|
| Earth | 1.3 cm (grape) | — |
| Moon | .35 cm (corn kernal) | 1 foot |
| Sun | 1.4 m (child's height) | 150 m (1 city block) |
| Jupiter | 15 cm (grapefruit) | 5 city blocks |
| Saturn | 12 cm (orange) | 10 city blocks |
| Uranus | 5.2 cm (apricot) | 20 city blocks |
| Neptune | 5 cm (plum) | 30 city blocks |
| Pluto | 0.23 cm (rice grain) | 40 city blocks |
| α Centauri | 1.6 m (small adult) | Earth's circumference |
| Center of Galaxy | — | 1.7 AU |
| Andromeda Galaxy | Sun – Pluto distance | 400 AU |
| Human | atom | — |

Energy and Luminosity (Power)

- Power and Luminosity are equivalent
- Power is Energy/Time, or Energy = Power \times Time
- Luminosity of the Sun: $L_{\odot} = 4 \cdot 10^{33}$ erg/s = $4 \cdot 10^{23}$ kw = $3 \cdot 10^{23}$ hp
- 1 watt = 10^7 erg/s
- 1 kilowatt = 10^3 watt
- 1 kilowatt-hour = 1 kwh = $3.6 \cdot 10^{13}$ erg
- 1 horsepower = 1 hp = 0.75 kw
- 1 BTU = $1.055 \cdot 10^{10}$ erg = 1.055 kw-second
- 1 megaton TNT = $3 \cdot 10^8$ kwh
- On Long Island, 1 kwh costs \$0.17
- The world's annual energy use is about 10^{28} ergs \Rightarrow $\$4.7 \cdot 10^{13} = \$47T$
- The world's power use thus averages $3.2 \cdot 10^{20}$ erg/s = 32 billion kw
- The Solar Constant is solar energy received per unit area per second at the Earth:
$$\frac{L_{\odot}}{4\pi(1 \text{ AU})^2} = 1.3 \cdot 10^6 \frac{\text{erg}}{\text{cm}^2 \text{ s}} = 0.13 \frac{\text{watt}}{\text{cm}^2}$$
- A collector of $2.4 \cdot 10^{14}$ cm² = 9300 mi² area operating at 100% efficiency is needed to continuously supply the world.

Mass

- Mass of the Sun: $M_{\odot} = 2 \cdot 10^{33} \text{ g}$
- Mass of the Earth:
 $M_{\oplus} = 6 \cdot 10^{27} \text{ g} = 1.3 \cdot 10^{25} \text{ lb.} = 6.6 \cdot 10^{21} \text{ tons} = 3 \cdot 10^{-5} M_{\odot}$
- Mass of Jupiter (J): $M_J = 1.9 \cdot 10^{30} \text{ g} = 318 M_{\oplus} = 0.95 \cdot 10^{-3} M_{\odot}$
- Mass–energy equivalence is stated in Einstein’s Law: $E = Mc^2$
- Energy equivalent of the Sun: $E_{\odot} = M_{\odot}c^2 = 2 \cdot 10^{54} \text{ erg}$
- Implied lifetime of the Sun:
 $\tau_{\odot} \simeq E_{\odot}/L_{\odot} = 5 \cdot 10^{20} \text{ s} = 1.7 \cdot 10^{13} \text{ yr} = 17 \text{ Gyr}$
(1 Gyr = 10^9 yr)
- Nuclear energy reactions in the Sun are only .007 (0.7%) efficient, and only the innermost 10% of the Sun will ever be “burned”:
 $\tau_{\odot} = 17,000 \times .007 \times .1 \text{ billion yrs} = 12 \text{ billion yrs}$

Mass Measurement

- Kepler's 1-2-3 Law

$$G(M_1 + M_2)P^2 = 4\pi^2(R_1 + R_2)^3$$

Measure orbital period P . If stellar velocity V_1 is measured (for example, from maximum Doppler shift of spectral lines) the circumference of M_1 's orbit is $2\pi R_1 = PV_1$. The total mass is

$$M_1 + M_2 = PV_1^3(1 + M_1/M_2)^3/(2\pi G).$$

If $M_2 \gg M_1$ then

$$M_2 = PV_1^3/(2\pi G).$$

Caveat: inclination angle should be known to find V_1 . If V_2 is also measured, then both M_1 and M_2 can be found.

- Gravitational Lensing
- Position in Hertzsprung-Russell Diagram
- Estimates Based on Luminosity Applied to molecular clouds and galaxies.