We propose a novel approach for studying neutrino oscillations with extragalactic neutrinos. We show that measurement of the neutrino induced upward hadronic and electromagnetic showers and upward muons could be used to detect $\nu_\mu \rightarrow \nu_\tau$ oscillations. We find significant signal to background ratios for the hadronic/electromagnetic showers with energies above 10 TeV to 100 TeV initiated by the extragalactic neutrinos. We demonstrate that a kilometer-size neutrino telescope has a very good chance of detecting neutrino oscillations.

Recent SuperK data on atmospheric neutrinos indicate $\nu_\mu \rightarrow \nu_\tau$ oscillations with mixing being nearly bi-maximal [1], in agreement with previously reported results on the atmospheric anomaly by Kamiokande [2] and MACRO [3] and consistent with limits from other experiments, e.g., CHOOZ [4]. Direct detection of $\nu_\tau$ appearance is extremely difficult because at low energies, the charged-current cross section for producing a tau is small and the tau has a very short lifetime. Several long-baseline experiments with accelerator sources of $\nu_\mu$ [5, 6, 7, 8, 9] have been proposed with the goal of detecting tau neutrinos from oscillations, thus confirming the SuperK results. Until recently, the only convincing evidence of neutrino oscillations involved indirect measurements, namely the disappearance of the expected neutrino fluxes from the sun and atmospheric neutrinos. A recent breakthrough in the study of neutrino oscillations came from the SNO experiment of a direct observation of solar neutrino conversion into other active flavor [10]. SNO measurement of the elastic scattering rate is consistent with the precision elastic scattering measurement by SuperK [11], and both experiments are in very good agreement with the theoretical prediction for the $^8$B flux [12].

We have recently discussed complementary way of detecting neutrino oscillations, namely the possibility of a kilometer-size neutrino telescope detecting neutrino oscillations using extragalactic sources of high-energy neutrinos such as Active Galactic Nuclei (AGN) and Gamma Ray Bursts (GRB) [13, 14, 15]. The large distances involved for astrophysical sources, on the order of one to thousands of Megaparsecs, make the next generation of neutrino experiments potentially sensitive to neutrino mass differences as low as $\Delta m^2 \sim 10^{-17}$ eV$^2$. Assuming maximal mixing, over such long baselines, half of the neutrinos arriving at the earth would be $\nu_\tau$'s in oscillation scenarios, the other half being $\nu_\mu$'s. The effect of attenuation of the neutrino flux due to interactions of neutrinos in the Earth is qualitatively different for $\nu_\mu$ and $\nu_\tau$ [16]. Muon neutrinos are absorbed by charged current interactions, while tau neutrinos are regenerated by tau decays. The Earth never becomes opaque to $\nu_\tau$, though the effect of $\nu_\tau \rightarrow \tau \rightarrow \nu_\tau$ interaction and decay processes is to degrade the energy of the incident $\nu_\tau$. The identical spectra of $\nu_\mu$ and $\nu_\tau$ incident on the Earth emerge after passage through the Earth with distinctly different spectra. The preferential penetration of $\nu_\tau$ through the Earth is of great importance for high energy neutrino telescopes such as AMANDA, NESTOR and ANTARES.

Here we consider $\nu_\mu$ and $\nu_\tau$ propagation through the Earth and show that the energy spectrum of the $\nu_\tau$ becomes enhanced at low energy. The degree of enhancement depends on the initial neutrino flux. We consider initial fluxes $F_\nu^0 \sim E^{-n}$ for $n = 1, 2, 3, 6$, a GRB flux [17] and an AGN flux [18]. We solve the coupled transport equations for lepton and neutrino fluxes as indicated below.

Let $F_{\nu_\tau}(E, X)$ and $F_\tau(E, X)$ be the differential energy spectrum of tau neutrinos and tau respectively at a column depth $X$ in the medium. Then, one can derive the following cascade equation for neutrinos as,

$$\frac{\partial F_{\nu_\tau}(E, X)}{\partial X} = - \frac{F_{\nu_\tau}(E, X)}{\lambda_{\nu_\tau}(E)} + \int_E^\infty dE_Y \left[ \frac{F_{\nu_\tau}(E_Y, X)}{\lambda_{\nu_\tau}(E_Y)} \right] \frac{dn}{dE} (\nu_\tau N \rightarrow \nu_\tau X; E_Y, E)$$
\[
+ \int_{E}^{\infty} dE_{y} \left[ \frac{F_{\tau}(E_{y}, X)}{\rho_{\tau}^{\text{dec}}(E_{y})} \right] \frac{dn}{dE} (\tau \rightarrow \nu_{\tau}X; E_{y}, E) + \int_{E}^{\infty} dE_{y} \left[ \frac{F_{\tau}(E_{y}, X)}{\lambda_{\tau}(E_{y})} \right] \frac{dn}{dE} (\tau \rightarrow \nu_{\tau}X; E_{y}, E)
\]

and for taus as,
\[
\frac{\partial F_{\tau}(E, X)}{\partial X} = - \frac{F_{\tau}(E, X)}{\lambda_{\tau}(E)} - \frac{F_{\tau}(E, X)}{\rho_{\tau}^{\text{dec}}(E, X, \theta)} + \int_{E}^{\infty} dE_{y} \left[ \frac{F_{\nu_{\tau}}(E_{y}, X)}{\lambda_{\nu_{\tau}}(E_{y})} \right] \frac{dn}{dE} (\nu_{\tau}N \rightarrow \tau X; E_{y}, E).
\]

The first term in Eq. (1) is a loss due to the neutrino interactions, the second is the regeneration term due to the neutral current, the third term is a contribution due to the tau decay and the last term is the contribution due to tau interactions. In Eq. (2), the first term is a loss due to tau interactions, the second term is a loss due to the tau decay, while the last term is a contribution from neutrino charged current interactions. As a practical matter, tau decays are more important than tau interactions at the energies considered here, though interactions become more important at higher energies [19]. Here \(\lambda(E)\) is the interaction length and \(\rho_{\tau}^{\text{dec}}(E, X, \theta)\) is the decay length for tau. The charged and neutral current energy distributions, \(dn/dE\), and the total cross section, \(\sigma_{\nu_{\tau}}^{\text{tot}}(E)\) are calculated taking into account recent improvements in our knowledge of the small-x behavior of the structure functions [20].

To demonstrate the importance of regeneration of tau neutrinos from tau decays, we evaluate the tau neutrino flux for several input neutrino spectra and compare to the attenuated \(\nu_{\mu}\) flux. For the incoming neutrino spectrum we use power law spectrum, i.e. \(F_{\nu}(E) \sim K \left( \frac{E}{T_{\nu}} \right)^{-n}\). In Figure 1 we show the energy dependence of the the ratio of fluxes for nadir angles \(\theta = 0^0\), \(\theta = 30^0\) and \(\theta = 60^0\) for \(\nu_{\mu}\) and \(\nu_{\tau}\).

![Figure 1: The energy dependence of the ratio of fluxes for nadir angles \(\theta = 0^0\), \(\theta = 30^0\) and \(\theta = 90^0\) for \(\nu_{\mu}\) and \(\nu_{\tau}\) assuming \(F_{\nu}(E) \sim E^{-n}\) with \(n = 1, 2\) and 3.6.](image)

For small nadir angles, \(\theta = 0\) and \(30^0\) and \(F_{\nu}(E) \sim 1/E\) we find that enhancement of tau neutrinos is in the energy range of \(10^2\) GeV and \(10^3\) GeV, while for \(\theta = 60^0\), the enhancement extends up to \(10^6\) GeV. In contrast the \(\nu_{\mu}\) flux is attenuated for all the nadir angles. When the incoming flux is steeper, \(n = 2\), the \(\nu_{\tau}\) flux appears to be attenuated at high energies, although
less than the $\nu_\mu$ flux. For $n = 3.6$, the energy dependence of these two fluxes is very similar, they are both reduced at high energies, and the effect is stronger for smaller nadir angle, since in this case the column depth is larger and there are more charged current interactions possible. In case of the AGN quasar model [18], for example, we find that the $\nu_\tau$ flux is a factor of 2 to 2.5 times larger than the input flux, for nadir angle, $\theta = 0$ and $E = 10^2 - 10^4$ GeV[13]. For larger angles, the effect is smaller.

The appearance of high energy tau neutrinos due to $\nu_\mu \rightarrow \nu_\tau$ oscillations of extragalactic neutrinos can be observed by measuring the neutrino induced upward hadronic and electromagnetic showers and upward muons. Charged-current interactions of the upward tau neutrinos below and in the detector, and the subsequent tau decay create muons or hadronic and electromagnetic showers. The background for these events are muon neutrino and electron neutrino charged-current and neutral-current interactions, where in addition to extragalactic neutrinos, we also include the background from atmospheric neutrinos.

In Figure 2a) we show that tau neutrinos give significant contributions to upward hadronic/EM showers, signaling the $\nu_\tau$ appearance. The solid lines show the event rates including contributions for $\nu_\tau$, $\nu_\mu$ and $\nu_e$. The dashed lines are from $\nu_\mu + \nu_e$, assuming equal neutrino and antineutrino fluxes of each flavor. We note that in the case of the $E^{-1}$ flux, the contributions from tau neutrinos are large, a factor of 4 times larger than the muon neutrino plus electron neutrino contribution at zero nadir angle. For horizontal showers, the enhancement factor is smaller, about 2 for all the energy thresholds that we consider. Similarly, for $E^{-2}$ flux, the tau neutrino contribution is a factor of 1.7 times larger than the muon neutrino plus electron neutrino contributions for upward showers. The rates for AGN quasar model [18], at zero nadir angle, are comprised of 60% tau neutrino induced events, decreasing to about 40% tau neutrino induced events for horizontal showers, translating to 25–80 shower events for $E_{\text{sh}}^{\text{min}} = 10$ TeV and 6–45 events for $E_{\text{sh}}^{\text{min}} = 100$ TeV with negligible atmospheric background [14].

We note that for the $E^{-1}$ flux, the shower event rates for $\nu_\tau + \nu_\mu + \nu_e$ are a factor of 3.3-3.7 larger than in the $\nu_\mu + \nu_e$ contributions in the no-oscillation scenario for $E_{\text{sh}}^{\text{min}} = 1 - 100$ TeV for $\theta = 0^\circ$. They are a factor of 1.6 enhanced for the horizontal shower rate. For the $E^{-2}$ flux, the enhancement is a factor of 1.4-1.6 relative to the $\nu_\mu + \nu_e$ no-oscillation shower rate for $E_{\text{sh}}^{\text{min}} = 1 - 100$ TeV. In the case of AGN models, if one assumes oscillations, the shower event rates are factor of 1.8-2.1 larger at zero nadir angle, decreasing to 1.5 for nearly horizontal showers [14]. Given the uncertainties in the normalizations of the extragalactic neutrino fluxes, combining muon rates and hadronic/EM rates offer the best chance to test the $\nu_\mu \rightarrow \nu_\tau$ oscillation hypothesis.

As concluded in earlier work [20, 21], in general, an energy threshold of between 10 TeV and 100 TeV for upward muons and showers is needed in order to reduce the background from atmospheric neutrinos. We find that diffuse AGN neutrino fluxes, as well as neutrinos from GRBs can be used to detect tau appearance. By measuring upward showers with energy threshold of 10 TeV, and upward muons, the event rates exceed the atmospheric background and are about a factor of 1.5–2 larger than in the no-oscillation scenario [14].

To determine the relative enhancement of the hadronic/EM signal compared to the muon signal, one can compare the rates for horizontal events, where tau neutrino pileup is small. For example, Figure 2b) shows a clear distinction between oscillation and no-oscillation scenarios, even in directions near horizontal, where there is no pileup. Even with a small tau neutrino pileup, the oscillation scenario can be distinguished from the no-oscillation scenario.

The detection of $\nu_\mu \rightarrow \nu_\tau$ oscillations with a point source might also be possible. With the resolution for the planned neutrino telescopes of 2$^\circ$, the atmospheric background is reduced by $3.8 \times 10^{-3}$. For upward showers, this gives less than 1 event per year for $E_{\text{sh}}^{\text{min}} = 1$ TeV, and even less for higher energy thresholds. Thus, if the point source has a flat spectrum, $F_{\nu_\mu \nu_\tau} = 10^{-16} E^{-1}$, then one would be able to detect tau neutrinos by measuring upward showers with $E_{\text{sh}}^{\text{min}} = 1$ TeV. In the more realistic case, when the point source has a steeper spectrum ($E^{-2}$), such as Sgr A$^*$ [22], a normalization of $10^{-7}/(\text{cm}^2\text{s sr GeV})$ would be sufficient for the detection of tau neutrinos with threshold of 1 TeV. Time correlations with variable point sources would further enhance the signal relative to the background.

We have demonstrated that extragalactic sources of neutrinos can be used as a very-long baseline experiment, providing a source of tau neutrinos and opening up a new frontier in studying neutrinos oscillations.
Figure 2: a) Hadronic/EM event rates as a function of nadir angle for $E_{\text{min}}^{\text{thr}} = 1\,\text{TeV}, 10\,\text{TeV}$ and $100\,\text{TeV}$. Hadronic/EM event rates from $\nu_\tau$ (solid line) compared hadronic/EM event rates from $\nu_\mu$ plus $\nu_e$ (dashed line) for $E^{-1}$ and $E^{-2}$. b) Ratio of Hadronic/EM event rate to muon event rate for the oscillation (upper shaded area) and no-oscillation (lower shaded area) scenarios as a function of nadir angle for threshold energies of $1\,\text{TeV}$, $10\,\text{TeV}$ and $100\,\text{TeV}$ for the indicated fluxes.

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References