You Can’t get There From Here

A View of Prospects for Space Travel
The Rocket Equation

Rockets work on the principle of Conservation of Momentum

A rocket of mass \( m \) and fuel mass \( \Delta m \) moving with a velocity \( v \) has momentum \((m + \Delta m)v\).

The rocket burns the \( \Delta m \) of fuel and ejects the exhaust at a velocity \( S \), with momentum \( \Delta m(v-S) \). The Rocket has a new momentum, \( m(v + \Delta v) \).

Conservation of momentum requires that

\[
(m + \Delta m)v = m(v + \Delta v) + \Delta m(v - S)
\]

which simplifies to

\[
\Delta v = S \frac{\Delta m}{m}
\]

You can integrate this equation to get what is known as the “Rocket equation”,

\[
V = S \ln\left(\frac{m_i}{m_f}\right)
\]

where \( m_i \) and \( m_f \) are the initial and final masses of the rocket (the difference is the amount of fuel burned), \( S \) is the ejecta velocity, and \( V \) is the final velocity of the rocket.

This works for single stage rockets. For multiple-stage rockets, you have to consider each stage separately and multiply the results.
Consequences of the Rocket Equation

\[ \frac{v}{S} = \ln \left( \frac{M_{\text{fuel}} + M_{\text{payload}}}{M_{\text{payload}}} \right) \]

\[ S \] = velocity of exhaust.
\[ v \] = final velocity of rocket.

Because you have to carry your fuel with you, the final velocity increases only logarithmically with the amount of fuel carried.
How Much Fuel Do You Need?

\[
\frac{M_{\text{fuel}}}{M_{\text{payload}}} \quad \frac{v}{S}
\]

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.7</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>2.3</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>4.6</td>
<td></td>
</tr>
</tbody>
</table>

The exhaust velocity $S$ depends on the type of fuel you burn.

<table>
<thead>
<tr>
<th>fuel</th>
<th>impulse</th>
<th>$S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{H}_2 + \text{O}_2$</td>
<td>3.2 kW hr/kg</td>
<td>3 km/s</td>
</tr>
<tr>
<td>nuclear fission</td>
<td>$2 \times 10^7$ kW hr/kg</td>
<td>8000 km/s</td>
</tr>
<tr>
<td>nuclear fusion</td>
<td>$2 \times 10^8$ kW hr/kg</td>
<td>25000 km/s</td>
</tr>
<tr>
<td>matter/antimatter</td>
<td></td>
<td>c</td>
</tr>
</tbody>
</table>
## The Cost to Accelerate

<table>
<thead>
<tr>
<th>Velocity</th>
<th>fuel</th>
<th>$\frac{m_{fuel}}{m_p}$ to accelerate</th>
<th>$\frac{m_{fuel}}{m_p}$ + to stop</th>
</tr>
</thead>
<tbody>
<tr>
<td>17,500 mph</td>
<td>chemical</td>
<td>14</td>
<td>185 (LEO)</td>
</tr>
<tr>
<td>25,000 mph</td>
<td>chemical</td>
<td>39</td>
<td>1521 (V$_{esc}$)</td>
</tr>
<tr>
<td>0.1c</td>
<td>fusion</td>
<td>2.3</td>
<td>5.4 65 million mph</td>
</tr>
<tr>
<td>0.5c</td>
<td>fusion</td>
<td>400</td>
<td>1600 $\gamma=1.155$</td>
</tr>
<tr>
<td>0.5c</td>
<td>matter/antimatter</td>
<td>0.6</td>
<td>1.7</td>
</tr>
</tbody>
</table>
Power and Acceleration. I.

1 gravity (g; 980 cm/s$^2$) is a comfortable acceleration

$$a = \frac{2P}{ms},$$

where $a$ is the acceleration, $P$ is the power, $m$ is the mass, and $s$ is the exhaust speed.

A chemical rocket requires $P > 1.5$ kW/kg

Burning $2\text{H}_2 + \text{O}_2 \rightarrow 2\text{H}_2\text{O}$ liberates 4 kW hr/kg.

To accelerate a mass $m$ at 1g requires burning $\text{H}_2+\text{O}_2$ at a rate of $10^{-3}$ m gm/sec.

The Saturn V burned 3000 tons of kerosene+$\text{O}_2$ per second.
Power and Acceleration. II.

To accelerate at g:
• A chemical rocket must generate 1.5 kW/kg
• A nuclear fusion rocket must generate 440 mW/kg
• A matter-antimatter rocket must generate 1.5 Gw/kg

• Power plants typically generate 0.5-1 Gw of power
The Speed Limit

186,000 MILES PER SECOND IS NOT JUST A GOOD IDEA

IT'S THE LAW

\[ c = 3 \times 10^5 \text{ km/s} \]
Special Relativity to the Rescue?

At low velocities \( (v \ll c) \),

- \( F=ma \) (Newton’s law)
- \( v=at \) (\( v = \)velocity; \( a = \)acceleration)
- \( E=\frac{1}{2}mv^2 \), or \( v = \sqrt{2E/m} \) (kinetic energy)

Einstein showed that all non-accelerated motion is relative, but that all observers agree on the value of \( c \). This requires some modifications to the Newtonian equations at high velocities.

- The dilation factor \( \gamma = \sqrt{1 - \frac{v^2}{c^2}} \)
- \( \gamma \) is close to one for \( v \ll c \), approaches \( \infty \) as \( v \to c \)
- At high velocities, time slows down. \( t = \gamma t_0 \)
- At high velocities, mass increases. \( m = \gamma m_0 \)
- \( v = \sqrt{2E/\gamma m} \)
At low velocities, under a constant acceleration $a$

$$v = a \cdot t \quad \text{(velocity)}$$

$$d = \frac{(at^2)}{2} \quad \text{(distance travelled)}$$

As $v$ approaches $c$, the speed of light, Newtonian theory breaks down and we need to use Einstein’s Theory of Special Relativity. Time dilation effects become important.

The time you experience $\tau$ slows down relative to that in the unaccelerated frame $t$.

$$\tau = t \sqrt{1 - \left(\frac{v}{c}\right)^2}$$

Under constant acceleration $a$, the formulae change to

$$v = \frac{at}{\sqrt{1 + \left(\frac{at}{c}\right)^2}}$$

$$d = \frac{c^2}{a} \left[\sqrt{1 + \left(\frac{at}{c}\right)^2} - 1\right]$$

The consequence of time dilation is that if you go very fast, you can get very far in a short period of proper time.
Time Dilation. II. Practice

<table>
<thead>
<tr>
<th>( \tau )</th>
<th>( t )</th>
<th>( v )</th>
<th>( d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elapsed time for traveller (years)</td>
<td>Elapsed time on Earth (years)</td>
<td>Maximum velocity ( c )</td>
<td>Distance travelled (light years)</td>
</tr>
<tr>
<td>1</td>
<td>1.01</td>
<td>0.25</td>
<td>0.06</td>
</tr>
<tr>
<td>2</td>
<td>2.08</td>
<td>0.46</td>
<td>0.26</td>
</tr>
<tr>
<td>3</td>
<td>3.29</td>
<td>0.64</td>
<td>0.59</td>
</tr>
<tr>
<td>4</td>
<td>4.70</td>
<td>0.76</td>
<td>1.08</td>
</tr>
<tr>
<td>5</td>
<td>6.41</td>
<td>0.85</td>
<td>1.77</td>
</tr>
<tr>
<td>10</td>
<td>24.2</td>
<td>0.99</td>
<td>10.3</td>
</tr>
<tr>
<td>20</td>
<td>296.8</td>
<td>0.9999</td>
<td>146</td>
</tr>
<tr>
<td>30</td>
<td>3613</td>
<td>( c )</td>
<td>1805</td>
</tr>
<tr>
<td>40</td>
<td>44,100</td>
<td>( c )</td>
<td>22,050</td>
</tr>
<tr>
<td>50</td>
<td>535,900</td>
<td>( c )</td>
<td>268,000</td>
</tr>
<tr>
<td>100</td>
<td>( 1.44 \times 10^{11} )</td>
<td>( c )</td>
<td>( 7.2 \times 10^{10} )</td>
</tr>
</tbody>
</table>

\( g = 9.8 \text{ m/sec}^2 = 1 \) Earth gravity
Possible Outs. 1.

Why carry your fuel? The **Bussard Ramjet**

- Space is not empty: about 0.1 H/cm$^3$
- Hydrogen is an excellent fuel

- To sweep up 1 gm of H, with
  - $v = 0.99c$
  - 100% efficiency
  requires a scoop radius of 40 km.
The atoms appear to be coming at you at 0.99c - or with 6 GeV (0.01 erg) rest energies

How do you stop them?
Possible Outs. II.

Why carry your fuel? **The Light Sail**

Works just like a sailboat, by conservation of momentum.

- $p = \frac{E}{c}$ (momentum carried by a photon)
- $a = \frac{2P}{mc}$ (acceleration; $P$=power)
- $P = \frac{LA}{4\pi d^2}$ ($L$=luminosity; $A$=sail area)

To accelerate a mass of 100 tons at 1 g requires $P=150,000$ GW, or a sail the size of a star, when you are a 2 light years out (half way between the stars.)
Possible Outs. III.

Why go all the way? Look for a **wormhole**.

The shortest distance in 3 dimensions may not be the shortest in 4 (or more) dimensions!

An *Einstein-Rosen bridge* is a wormhole connecting two different universes.

**Do wormholes exist?**
- Solutions of General Relativity permit them.
- They are unstable (barring exotic matter with a negative energy density).
- They require a “white hole” on the other end, which violates the second law of thermodynamics.
Possible Outs. IV.

**Warping space.**

- You can exceed $c$ **globally**; you cannot exceed it **locally**.
- If you can make space *contract* ahead, and *expand* behind, your local space can move with an arbitrarily high velocity.

**Warping space requires:**
- An awful lot of energy
- Exotic particles with negative energy
- Negative gravity

**But you get:**
- Arbitrarily fast speeds
- No time dilation
- No acceleration
- No causal paradoxes
Conclusions

- 1g accelerations are convenient for human space travel.

- Chemical fuels can provide this acceleration, but the small $S$ means that $m_i/m_f$ is prohibitive.

- Nuclear fusion provides better mass ratios, at the cost of low acceleration.

- Matter-antimatter provides the best mass ratios, but requires the most power.

- At present, there is no reasonable expectation of travel at 1g accelerations for significant distances.

Space is big; space travel is slow.