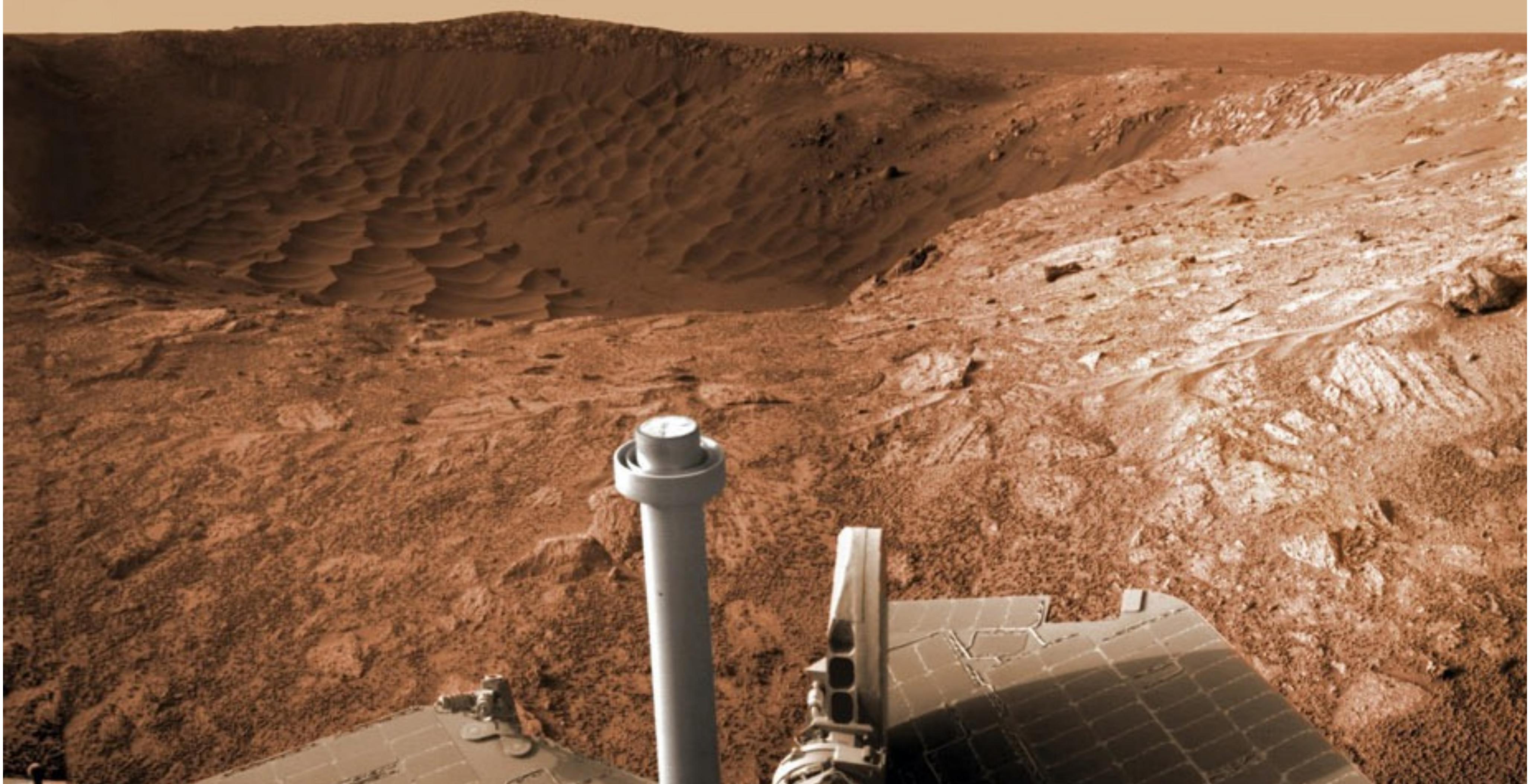
Astronomy Basics



Large and Small Numbers

Astronomers work with very large and very small numbers.

For example:

- The **radius** of the **sun** is 70,000,000,000 centimeters
- The **mass** of the **sun** is 20,000,000,000,000,000,000,000,000,000,000,000 grams
- The **radius** of a **Hydrogen atom** is 0.000,000,01 centimeters
- The **mass** of a **Hydrogen atom** is 0.000,000,000,000,000,000,000,001,6 grams

Such numbers are at best inconvenient to use

Scientific Notation

Scientists use a shorthand called **scientific notation**

Any number can be expressed as the product of two other numbers.

Usually, one of the numbers is a power of 10.

For example,

- $200 = 2 \times 100$ or $2 \times 10 \times 10$, or 2×10^2.
- $70{,}000{,}000{,}000 = 7 \times 10{,}000{,}000{,}000 = 7 \times 10^{10}$

Here, the 10 is called the ***exponent***.

The **exponent** is the number of zeros which follow the initial number called the **mantissa**.

Scientific Notation

Numbers with absolute values less than 1:

The exponent is negative, because $10^{-n} = 1/10^n$.

To get the **exponent**, count the number of zeros to the right of the decimal place, add 1, and then take the negative.

For example, $0.002=2 \times 0.001=2 \times 10^{-3}$.

Numbers between 1 and 10 have exponents of 0 because $10^0=1$.

Significant Figures

$1.234 \ x \ 10^6 = 12.34 \ x \ 10^5 = 0.1234 \ x \ 10^7$

By convention, we use a **mantissa** with one figure to the left of the decimal place ($1.234 \ x \ 10^6$ is preferred).

The significant figures are the number of digits in the **mantissa.**

This number ($1.234 \ x \ 10^6$) has 4 significant figures.

For most purposes in this course, **2 or 3 significant figures suffice.**

(In Engineering notation, the mantissa is a multiple of 3)

Significant Figures: Example

The Astronomical Unit au = 1.495978707×10^8 km (10)

The speed of light c = 2.99792458×10^5 km/s (9)

For the purposes of this course

c = 3×10^5 km/s (1)

au = 1.5×10^8 km (2)

Recall that distance / velocity = time

The average light-travel time to the Sun is 1 au/c, =

499.004811 seconds (9 significant figures)

500 sec (1 significant figure)

Is the difference important?

Note: the actual distance from the Earth to the Sun varies by 1% over the year

Manipulating Numbers

Scientific notation simplifies manipulation of large and small numbers.

Suppose you wanted to determine the *moment of inertia* of the Sun. This is the product MR^2.

$I = 2x10^{33}\ x\ 7x10^{10}\ x\ 7x10^{10}$ Note: I am ignoring the dimensions

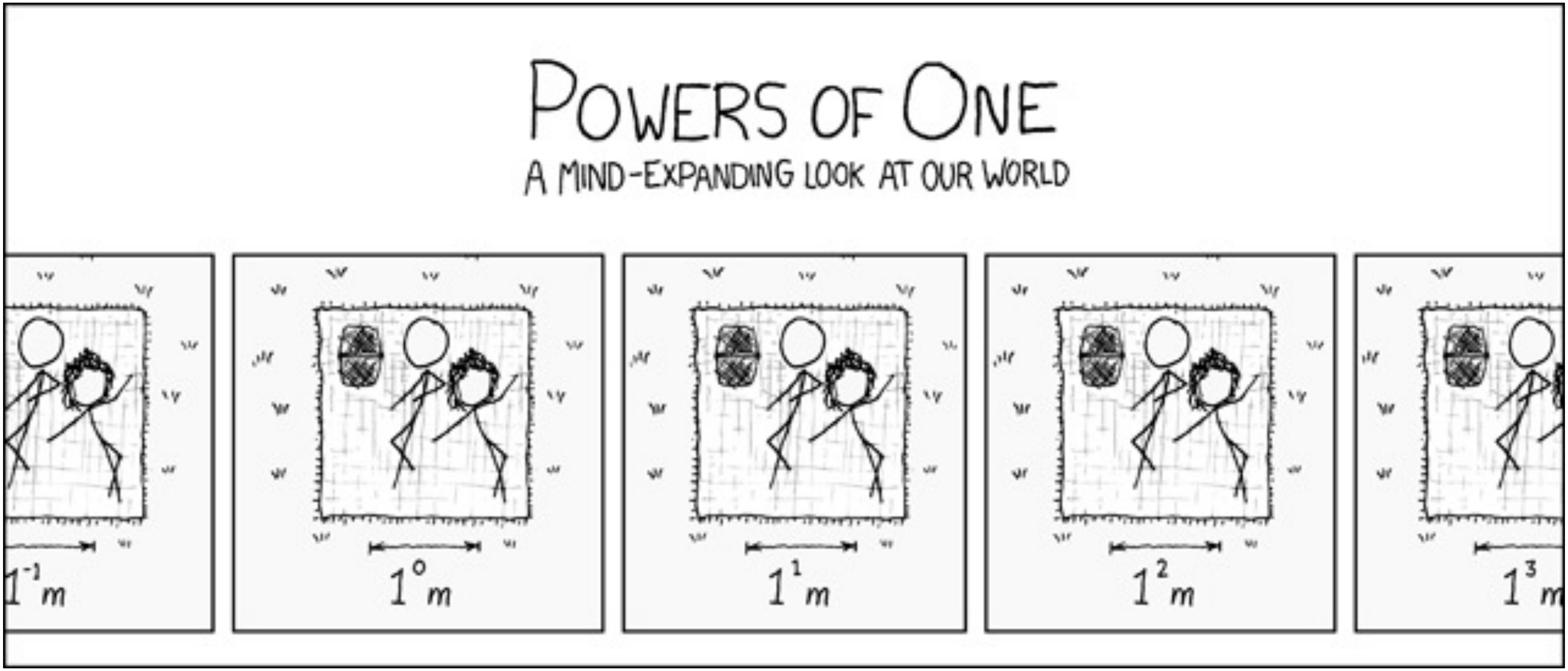
Multiply the **mantissas** ($2x7x7 = 98$)

Add the **exponents** ($33+10+10 = 53$)

$I=98\ x\ 10^{53} = 9.8\ x\ 10^{54}$, or $1.0\ x\ 10^{55}$

For division, subtract the exponents

[redacted]


POWERS OF ONE
A MIND-EXPANDING LOOK AT OUR WORLD
1^{-1}m
1^{0}m
1^{1}m
1^{2}m
1^{3}m

xkcd.com/271/

Dimensional Analysis

- We deal with **quantities**, not pure numbers.
- Quantities have units (e.g., km, sec)
- Units need to be included in your calculations.
- e.g.; speed has units of km/s or miles/hr
  - You calculate speed by dividing a quantity with units of **distance** by one with units of **time**.
  - If the units are not correct, you are probably doing something wrong!
- Previous example:
  - Moment of inertia $I = MR^2$
  - $I = 2x10^{33}$ gm x $7x10^{10}$ cm x $7x10^{10}$ cm = 9.8×10^{54} gm cm^2

Dimensional Analysis. II. A Simple Example

Suppose you forget the relationship between distance, time, and velocity.

- Write down the dimensions of the quantities:
  - Distance has units of [cm] (or km, or miles, or feet…)
  - Time has units of [seconds] (or years …)
  - Velocity has units of [km/hr] (or cm/sec …)
- To make the dimensions work out, the relation must be
  - Velocity=distance/time ([cm/s]). Or,
  - Distance = velocity *x* time ([cm/s *x* s] = [cm])
  - Time=distance/velocity ([cm/ cm/s] = [s])

Dimensional Analysis. III. Why this is important

Mars Climate Observer

Arrived at Mars November 1999

Mission was normal to this point

Failed to enter orbit: hit the atmosphere and burned up

Cause:

Thruster output in units of pounds of force [lb g] ($g=32\ ft/s^2$)

Expected output in Newtons [km m/s^2]

1 lb = 4.45 N

Result: trajectory off by 105 mi after nearly 3.7×10^8 mi

Further information and links:

https://www.wired.com/2010/11/1110mars-climate-observer-report/

Basic Units of Measurement

Distance

Centimeter (cm): *1,650,763.73 wavelengths in vacuum of the radiation corresponding to the transition between the levels $2p^{10}$ and $5d^{5}$ of the krypton^{86} atom.*
Meter (m): *100 cm. Originally 10^{-4} of the mean distance from the North Pole to the equator.*
Kilometer (km): *10^{3} m.* **Megameter (Mm):** *10^{6} m.*
Angstrom (Å): *10^{-10} m.*
Nanometer (nm): *10^{-9} m.*
Micron (μm): *10^{-6} m.*
Inch: *2.54 cm;* **Mile:** *1.62 km*

Astronomical Distances

Solar radius ($R_{\odot}$): *7×10^{10} cm*
Astronomical Unit (au): *Earth-Sun distance=1.5×10^{13} cm*
Light Year (ly): *9.46×10^{17} cm*
Parsec (pc): *3.26 ly*

Angular Measures

- There are 360 **degrees** (^{o}) in a circle
- There are 60 **minutes of arc** (‘) in a degree
- There are 60 seconds of **arc** (“) in a minute of arc
- The arc from the horizon to the zenith is 90^o

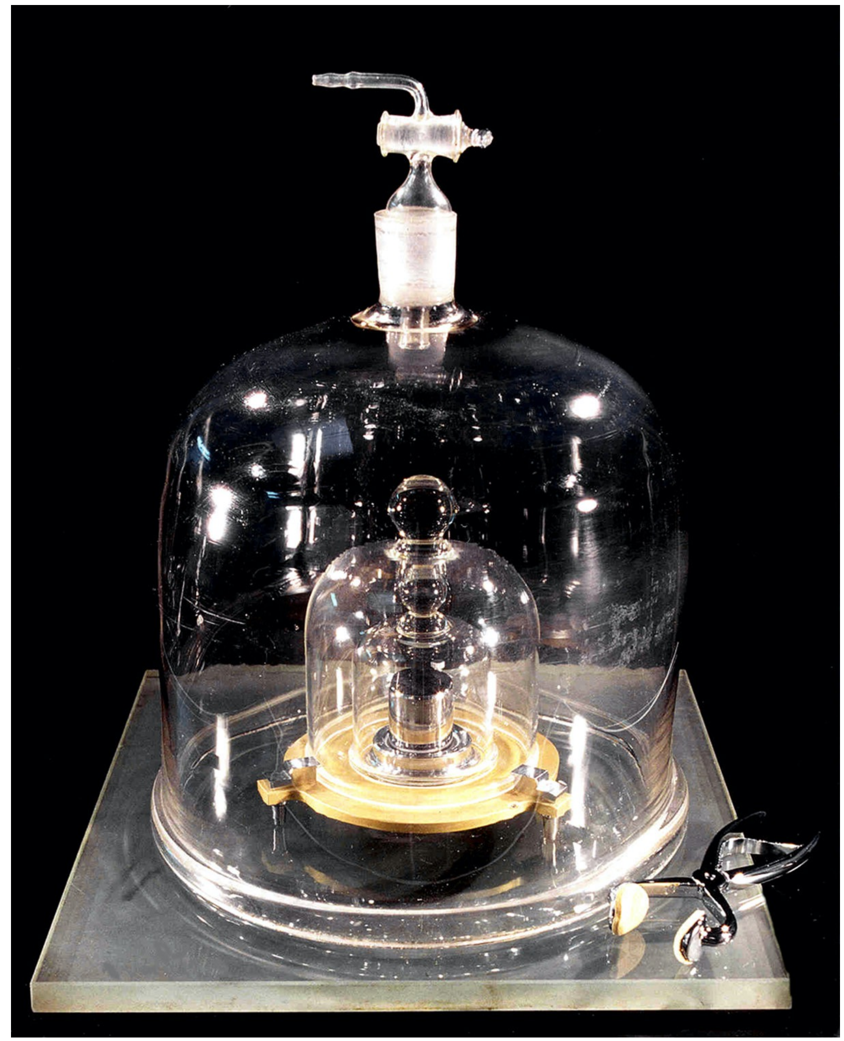
- At arm’s length,
  - The tip of your thumb to the tip of your little finger subtends about 20^o
  - Your closed **fist** subtends about 10^o
  - Your **knuckles** subtend about 2^o
  - A **finger** is about 1^o wide

- The Moon and Sun are about ½ ^{o} in diameter

Mass

gram (g): *defined such that the Planck Constant $h = 6.62607015 \times 10^{-34}\ kg \cdot m^2 \cdot s^{-1}$.* (as of 20 May 2019)

Formerly defined by the mass of a platinum-iridium cylinder (le grande K) stored in a vault in Paris.



kilogram (kg): *1000 gm*

Pound (lb): *454 gm*

Stone: *14 lb*

Earth mass ($M_\oplus$ or M_E): *$6x10^{27}$ gm*

Solar mass ($M_\odot$): *$2x10^{33}$ gm*

Time

Second (s): *the duration of 9,192,631,770 periods of the radiation corresponding to the transition between two hyperfine levels of the ground state of cesium^{133}.* ($E=h\nu$)

Day (d): *86,400 seconds. Mean rotation of the Earth*

Solar year (yr): *365.2422 days. Approx $\pi \times 10^7$ s. Orbital period of Earth.*

Platonic Year*: 25,800 yrs. Earth-Sun precession period.*

Galactic Year: *~250 million years. Time for Sun to orbit the Galactic Center*

Velocity

c (speed of light): 299,792 km/s (3x10^5 km/s)

Sun orbiting the Galaxy: 220 km/s

Earth orbiting the Sun: 30 km/s

Low Earth orbit: 7.6 km/s (27,500 km/h)

Moon orbiting Earth: 0.6 km/s (2200 km/h)

Temperatures

scale	Absolute zero	Water freezes	Water boils
Kelvin **(K)**	0	273.15	373.15
Celsius **(C)**	-273.15	0	100
Fahrenheit **(F)**	-456	32	212

$1K = 1C^o = 9/5\ F^o$; $C=(F-32)*5/9$