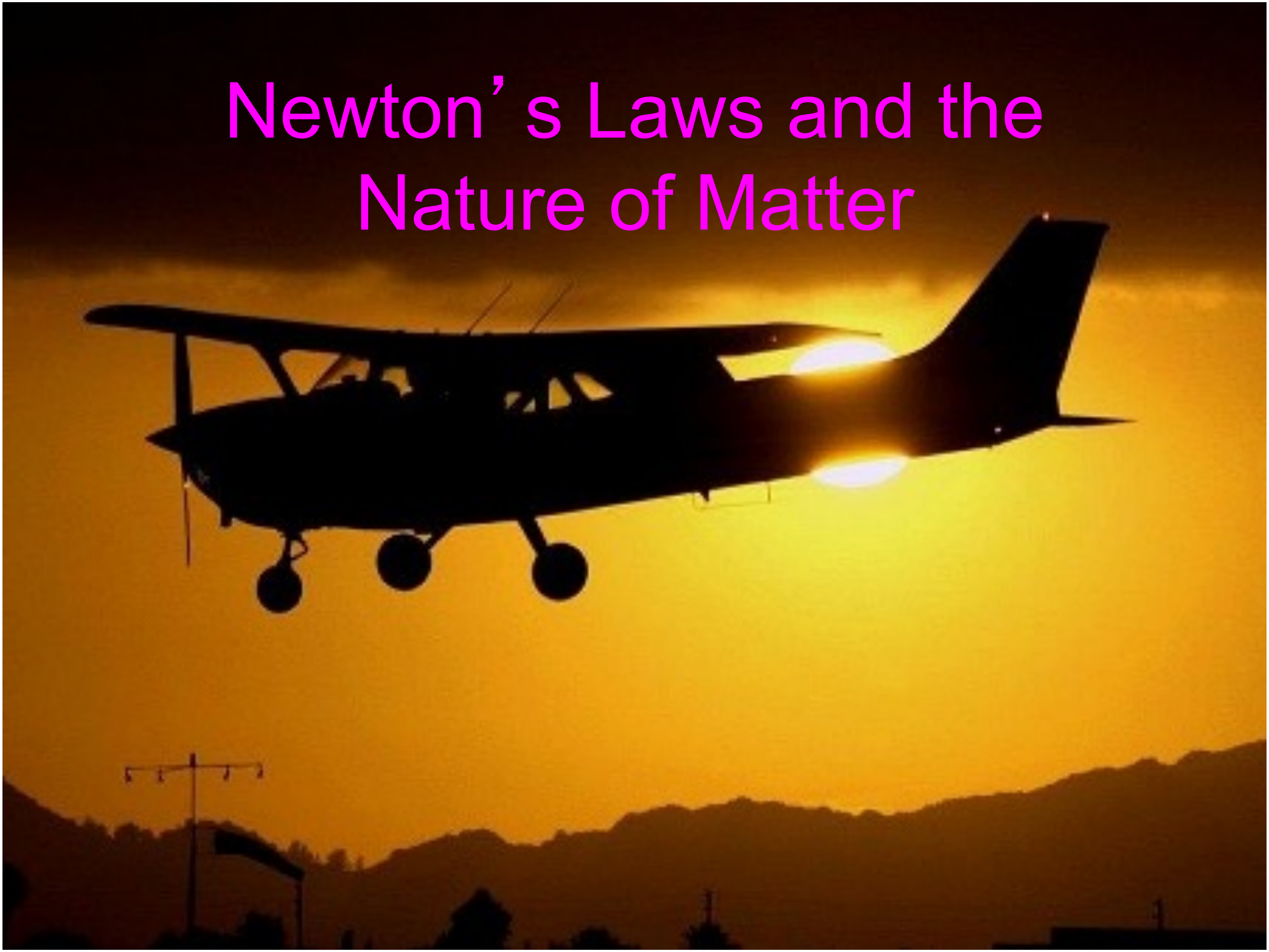
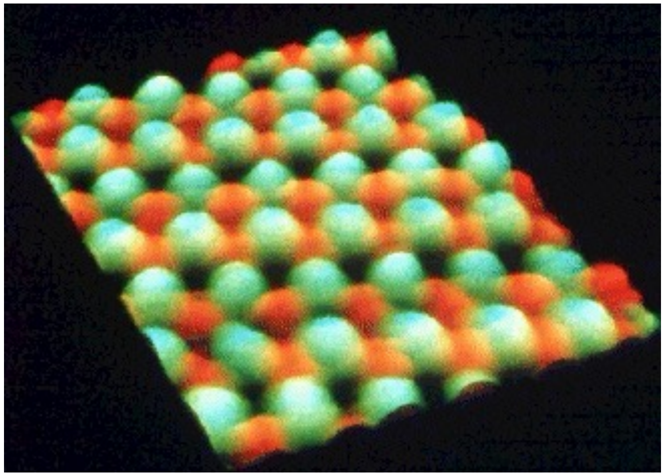
Newton's Laws and the Nature of Matter



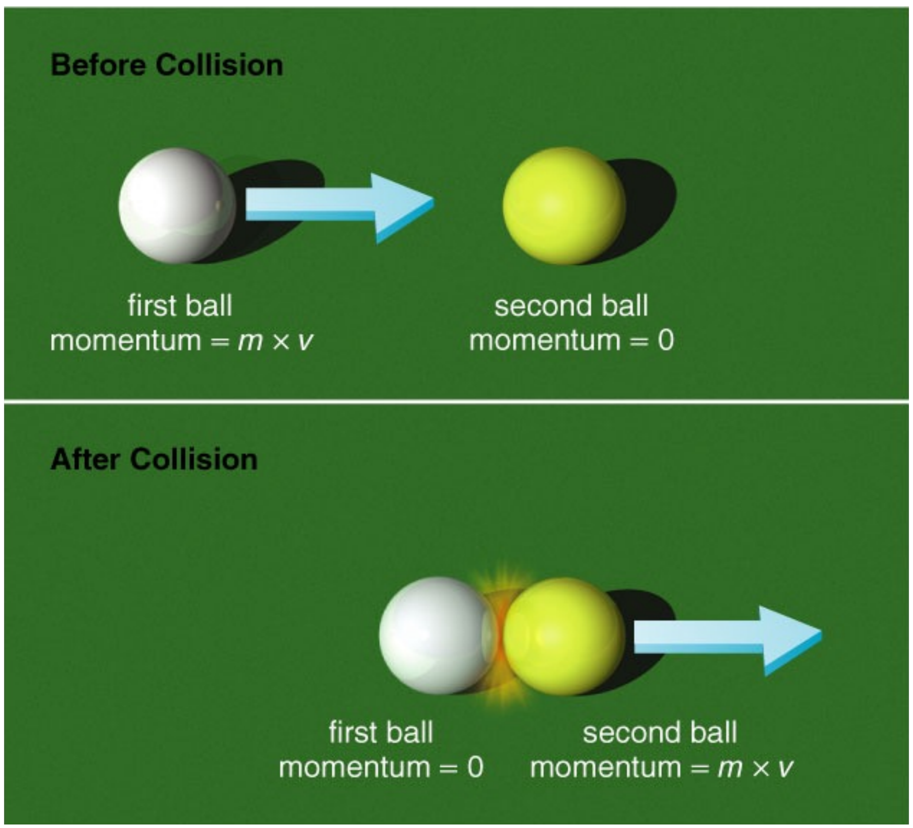
The Nature of Matter

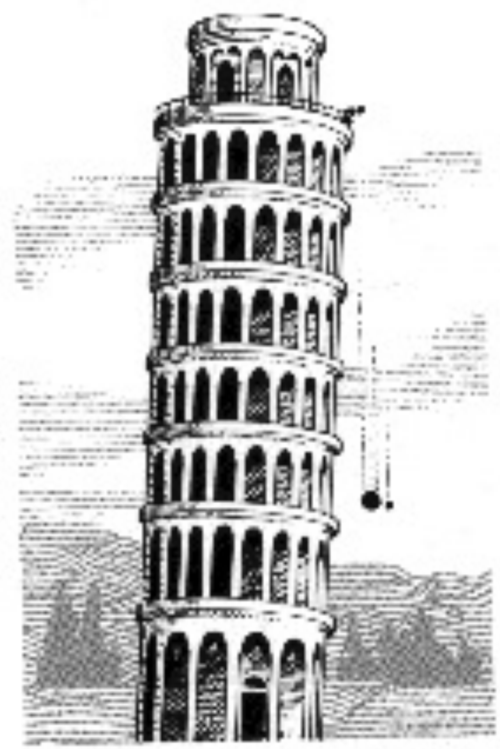
- Democritus (c. 470 - 380 BCE) posited that matter was composed of **atoms**
- *Atoms*: particles that can not be further subdivided
  - Not really true – there are 2 further levels of organization
- 4 kinds of atoms: *earth, water, air, fire* (the Aristotelian elements)



Bulk Properties of Matter

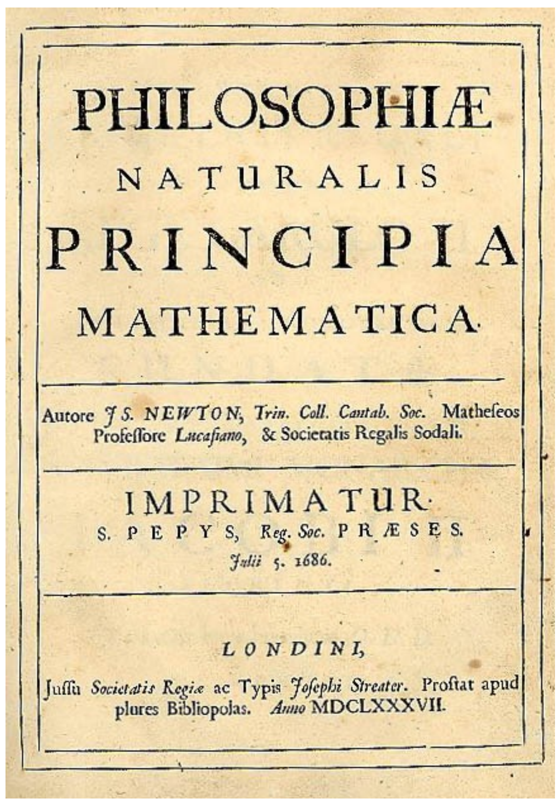
- Galileo showed that momentum (mass *x* velocity) is conserved
- Galileo experimented with inclined planes
- Observed that different masses fell at the same rate





Isaac Newton

- Quantified the laws of motion
- Invented modern kinematics
- Invented calculus
- Experimented with optics
- Built the first reflecting telescope



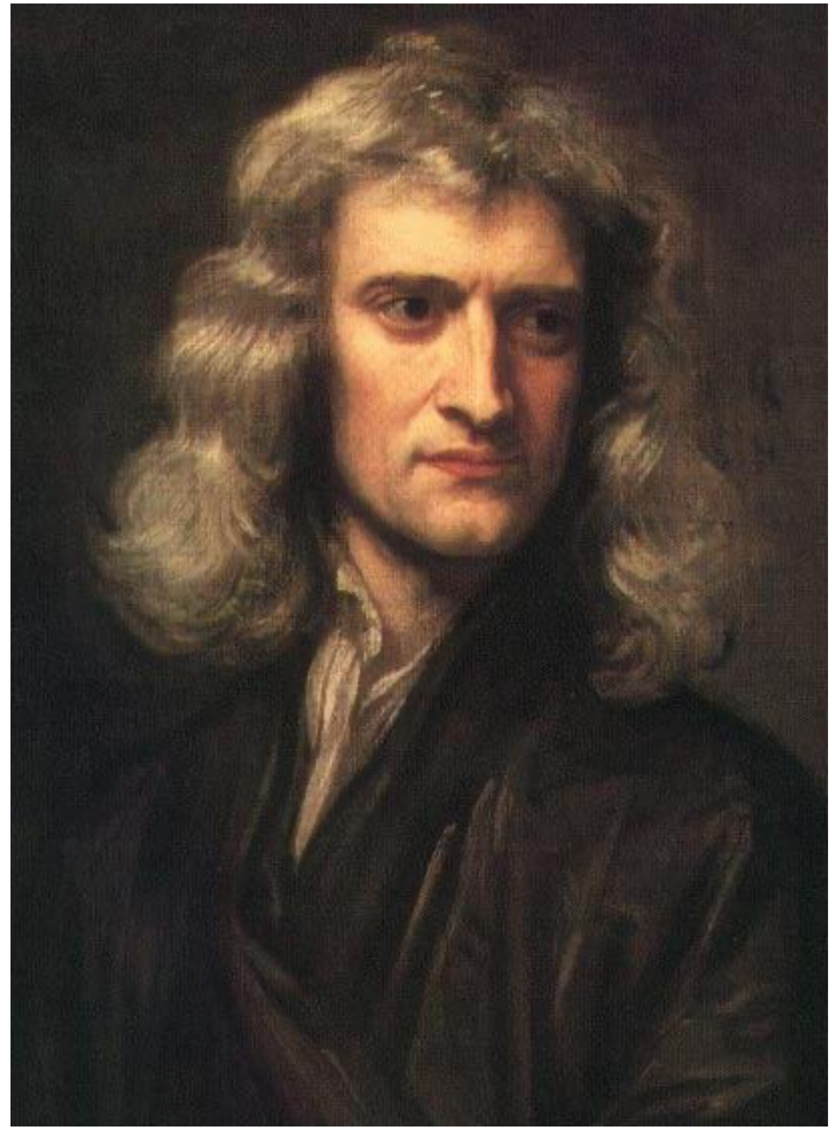
PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA

Autore *J S. NEWTON*, *Trin. Coll. Cantab. Soc.* Matheſeos Profeſſore *Lucaſiano*, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, *Reg. Soc.* PRÆSES.
Julii 5. 1686.

LONDINI,

Juſſu *Societatis Regiæ* ac Typis *Joſephi Streater*. Proſtat apud plures Bibliopolas. *Anno* MDCLXXXVII.



(1642-1727)

Newton's Laws

I. An object in motion remains in motion, or an object at rest remains at rest, unless acted upon by a force

This is the law of conservation of momentum, **mv = constant**

Newton's Laws

II. A force acting on a mass causes an acceleration.

F = ma

Acceleration is a change in velocity

Newton’s Laws

III. For every action there is an equal and opposite reaction

$$m_1a_1 = m_2a_2$$

Forces

Newton posited that gravity is an attractive **force** between two masses **m** and **M.** From observation, and using calculus, Newton showed that the force due to gravity could be described as

$$\mathbf{F_g = G\ m\ M\ /\ d^2}$$

G, the gravitational constant = 6.7×10^{-8} cm^3 / gm / sec

Gravity is an example of an *inverse-square law*

Forces

By Newton's second law, the gravitational force produces an *acceleration*. If **M** is the gravitating mass, and **m** is the mass being acted on, then

$$F = ma = G\,m\,M / d^2$$

Since the mass **m** is on both sides of the equation, it cancels out, and one can simplify the expression to

$$a = G\,M / d^2$$

Newton concluded that the gravitational acceleration was *independent of mass*.

An apple falling from a tree, and the Moon, are accelerated at the same rate by the Earth.

Galileo was right; Aristotle was wrong. A feather and a ton of lead will fall at the same rate.

Apollo 15

Hammer and Feather

Forces and the 3rd Law

$F = ma = G\,m\,M / d^2$

If you are **m** and **M** represents the mass of the Earth, **a** represents your downward acceleration due to gravity.

Your **weight** is the upward force exerted on you by the surface of the Earth.

The **gravitational force** down and your **weight** balance and you do not accelerate. *You are in equilibrium*.

Suppose you use **M** to represent your mass, and **m** to represent the mass of the Earth. Then, **a** is the acceleration of the Earth due to your mass. This is small, but real. *Your acceleration is some 10^{21} times that experienced by the Earth.*

Orbits

Orbit: the trajectory followed by a mass under the influence of the gravity of another mass.

Gravity and Newton's laws explain orbits.

In circular motion the acceleration is given by the expression $\mathbf{a=V^2/d}$ where **V** is the velocity and **d** is the radius of the orbit.

This is the centrifugal force you feel when you turn a corner at high speed: because of Newton's first law, you want to keep going in a straight line. The car seat exerts a force on you to keep you within the car as it turns.

Orbital Velocity

The acceleration in orbit is due to gravity, so

$$V^2/d = G M / d^2$$

which is equivalent to saying

$$V = \sqrt{(GM/d)} .$$

This is the velocity of a body in a circular orbit.

In low Earth orbit, orbital velocities are about 17,500 miles per hour.

If we know the orbital velocity **V** and the radius of the orbit **d**, then we can determine the mass of the central object **M**.

This is the ***only*** *direct way to determine the masses of stars and planets.*

What Keeps Things in Orbit?

There is no mysterious force which keeps bodies in orbit.

Bodies in orbit are continuously falling. What keeps them in orbit is their *sideways (transverse) velocity*. The force of gravity changes the direction of the motion by enough to keep the body going around in the orbit.

An astronaut in orbit is **weightless** because he (or she) is continuously falling.

Weight is the force exerted by the surface of the Earth to counteract gravity.

The Earth, the Sun, and the Moon have no weight!

Your weight depends on where you are - you weigh less on the top of a mountain than you do in a valley;

Your mass is not the same as your weight.

Newtonian Mechanics

Newton's laws, plus the law of gravitation, form a theory of motion called **Newtonian mechanics.**

It is a theory of masses and how they act under the influence of gravity.

Einstein showed that it is incomplete, but it works just fine to predict and explain motions on and near the Earth.

A more complete (and complex) theory of space and mass is Einstein's **General Theory of Relativity**.

Conservation Laws

- **Energy** is conserved
  - Energy can be transformed
- **Linear Momentum** is conserved

  M *x* v
- **Angular Momentum** is conserved

  M *x* v *x* d

Energy

We are concerned with two kinds of energy in astronomy:

- **kinetic energy** (abbreviated **K**)
- **potential energy** (abbreviated **U**)

Kinetic energy is energy of motion: $\mathbf{K = \frac{1}{2}\, m\, V^2}$

Kinetic energy can never be negative.

Potential energy is energy that is available to the object, but is currently not being used.

The potential energy due to gravity is $\mathbf{U = -G\, m\, M / d}$

As a body falls due to gravity, its potential energy decreases and its kinetic energy increases.

Energy is conserved, so the sum of K and U must stay constant.

Deriving Kepler's 3^{rd} Law

$P^2 = d^3$ (Kepler's 3^{rd} law, P in years and d in au)

$V = \sqrt{(GM/d)}$ (from Newton)

The circumference of a circular orbit is $2\pi d$.

The velocity (or more correctly, the **speed**) of an object is the distance it travels divided by the time it takes, so the orbital speed is **$V_{orb} = 2\pi d/P$**

Therefore **$\sqrt{(GM/d)} = 2\pi d/P$**

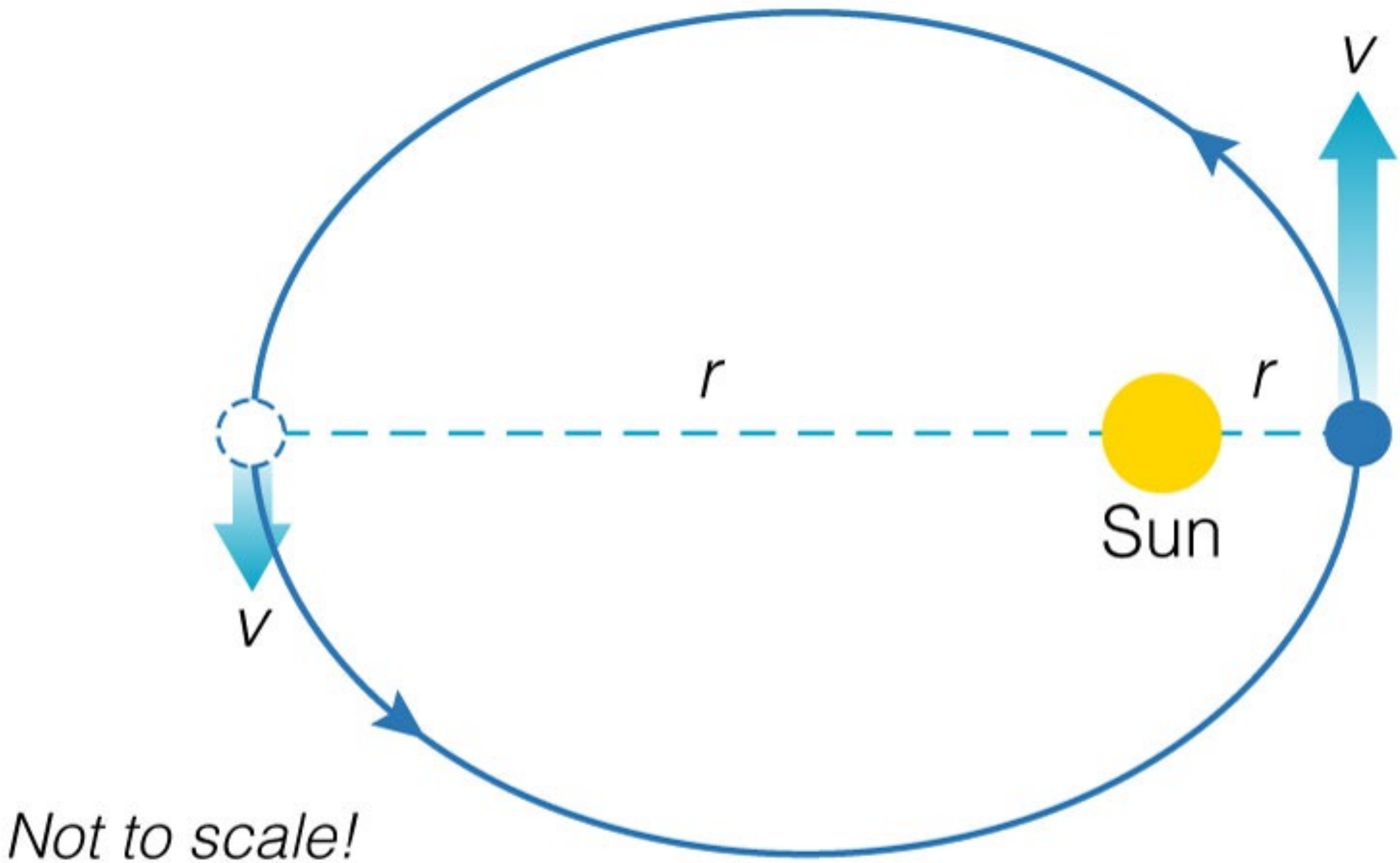
Square both sides: **$GM/d = 4\pi^2 d^2/P^2$**

⇨ **$P^2 = (4\pi^2/GM)d^3$**

$4\pi^2/GM$ = 1 $year^2/au^3$, or 2.96×10^{-25} $seconds^2 / cm^3$

This holds not only in our Solar System, but everywhere in the universe!

Deriving Kepler's 2nd Law



Motivating Kepler's 2^{nd} Law

You can use either conservation of energy, or conservation of angular momentum

In orbit, **K+U** is a constant (and is less than zero)

If the planet gets closer to the Sun,

- **d** decreases
- the potential energy **U** (= $-G\ m\ M\ /\ d$) decreases
- **K** ($\frac{1}{2}\ mV^2$) must increase.

The orbital speed must increase.

By conservation of angular momentum, mvd is constant.

Orbital velocities are faster closer to the Sun, and slower when further away.

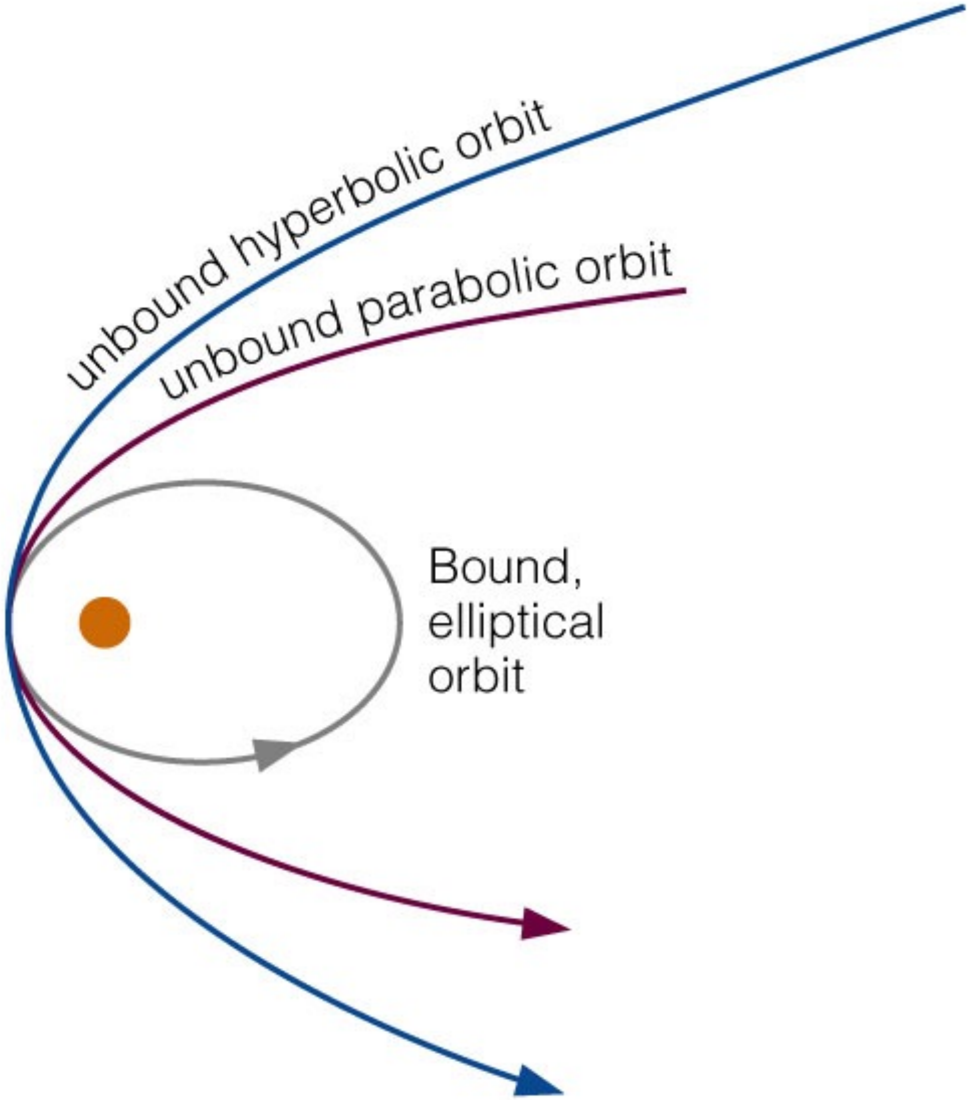
Orbital Energy

Orbits with negative total energy are bound.
Circular and elliptical orbits are bound.

If **K=-U**, the total energy is 0.
The trajectory is parabolic.
This gives the escape speed, $\mathbf{V_{esc} = \sqrt{(2G\ M\ /\ d)}}$

If **K > -U**, the trajectory is hyperbolic.

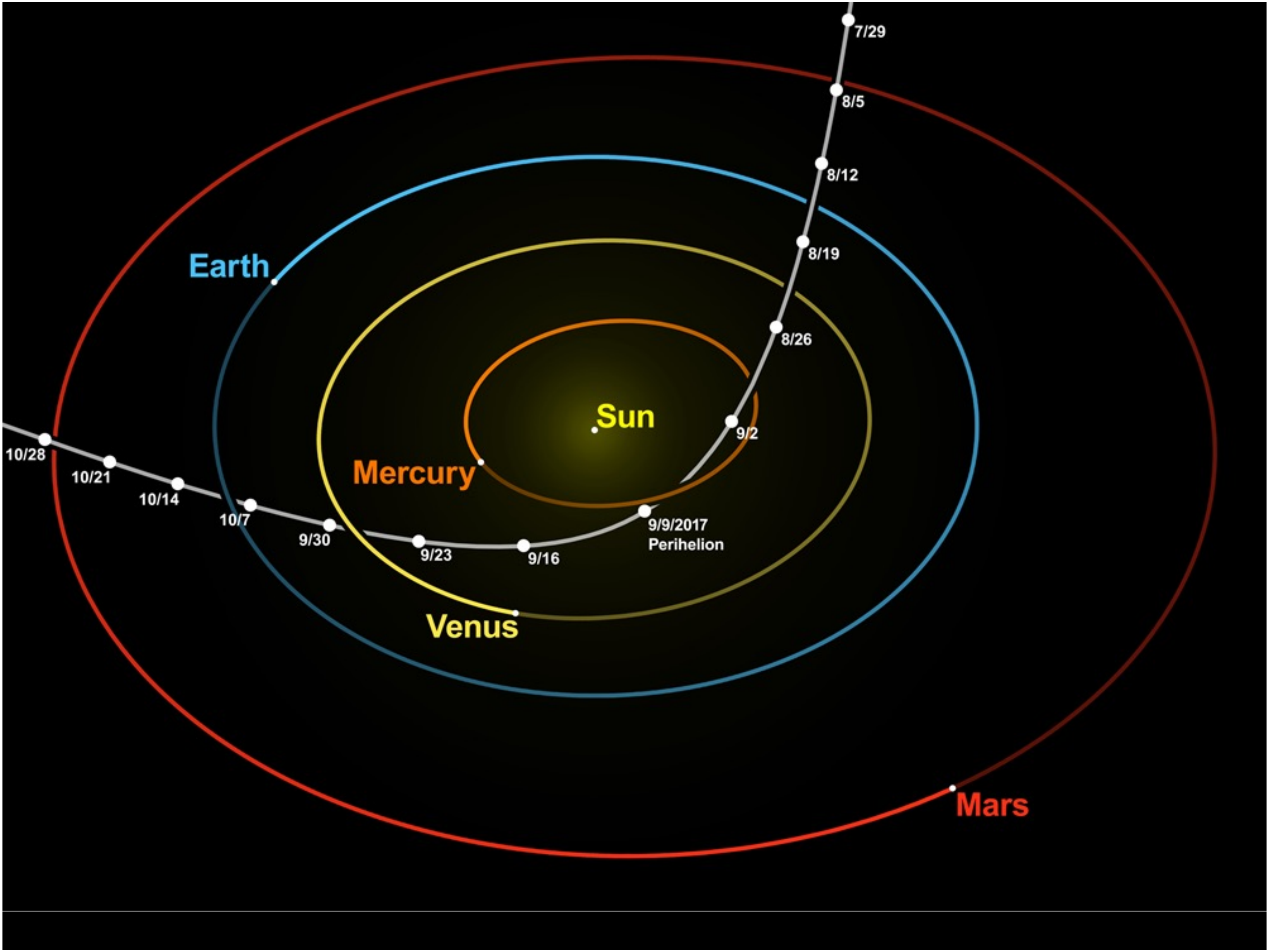
Orbital Energy



K+U > 0

K+U = 0

K+U < 0


7/29
8/5
8/12
8/19
Earth
8/26
Sun
9/2
Mercury
10/28
10/21
10/14
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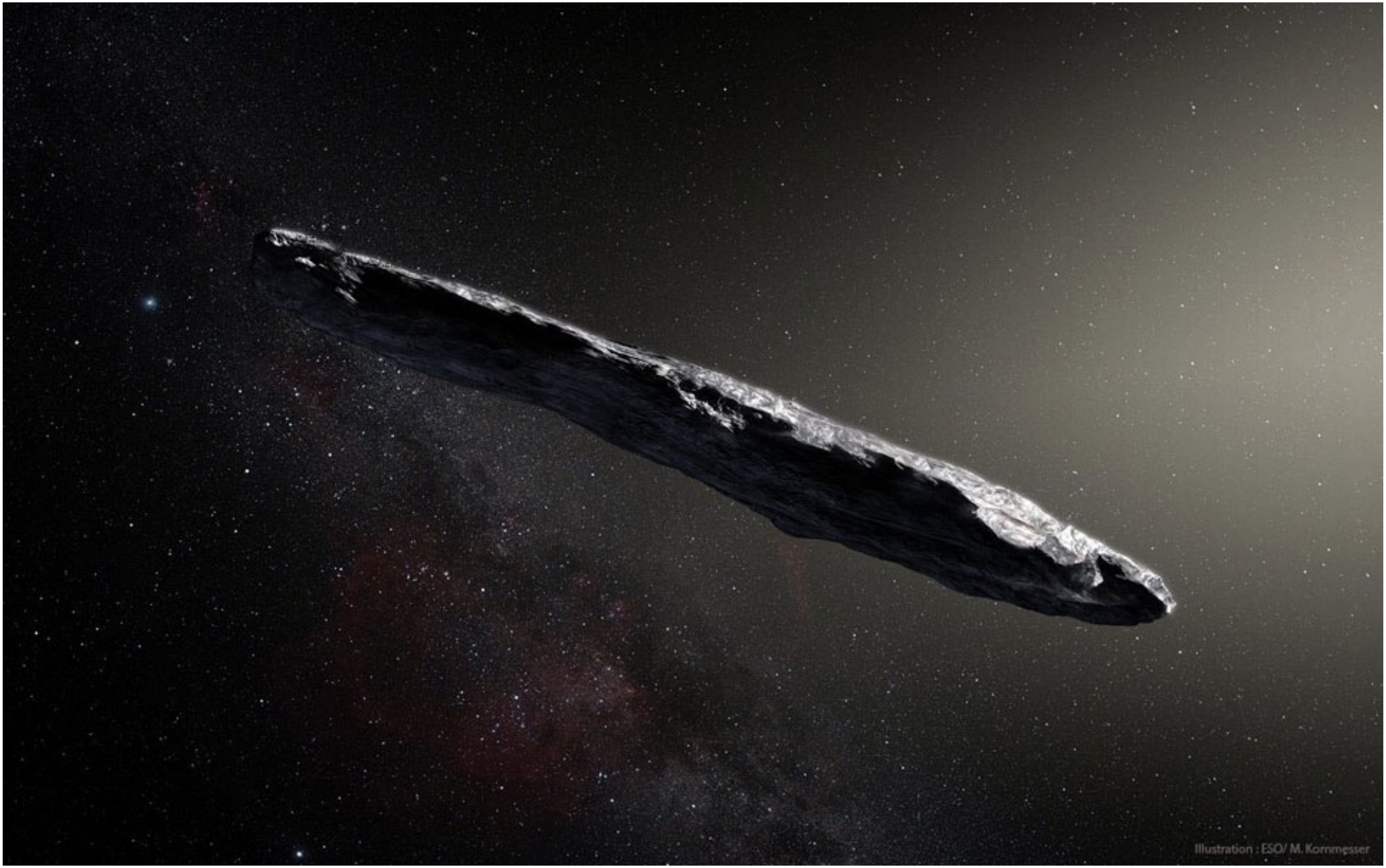
9/23
9/16
9/9/2017
Perihelion
Venus
Mars

‘Oumuamua



Hyperbolic orbit – a visitor from outside the solar system