Components of Galaxies – Stars

General Ref: B&M, pg 87–111, 123–126, S5.1-5.1.9

What properties of stars are important for our understanding of galaxies?

1) Temperature - Determines the wavelength range over which the radiation is emitted.

2) Chemical Composition (metallicities)

3) Lifetimes - Determines the timescale over which a particular type of star might affect the overall properties of the galaxy

4) Mass

5) Luminosity

1. Temperature

\[ T_e^4 = \frac{L_{\text{star}}}{4\pi\sigma R_{\text{star}}^2} \]

where \( T_e \) is the effective temperature - the temperature of a blackbody having the same radiated power per unit area, \( L_{\text{star}} \) and \( R_{\text{star}} \) are the luminosity and radius of the star and \( \sigma \) is the Stefan-Boltzmann constant

[Plot of blackbody temperature vs. real temperature for sun, and intensity vs. \( \lambda \) for blackbodies of different temperatures.]

2. Chemical Composition (metallicity)

- Older stars have higher scale heights and low metal abundance
- Younger stars have smaller scale heights and higher metal abundance

  - This is because the older stars are created from gas that has yet to be enriched by metals produced from Supernovae. Also, old stars were formed before much dissipation had occurred, so their scale heights are higher.

  [plot of globulars in Milky Way with different metal abundances]

There are two ways in which metallicity is designated:

1) \([\text{Fe/H}] = \log[n(\text{Fe})/n(\text{H})] - \log[n(\text{Fe})/n(\text{H})]_{\odot} \]

where \( n(X) \) is the number density of \( X \).
2) $X = H$, $Y = He$, and $Z = \text{others}$. For solar metallicities, $X = 0.70$, $Y = 0.28$, and $Z = 0.02$.

3. The Hertzsprung-Russell (HR) Diagram or Color-Magnitude Diagram

A useful way to visualize the properties of stars is to use the Hertzsprung-Russell (HR) Diagram or Color-Magnitude Diagram

1) Theoretical Diagram = Luminosity vs. Effective Temperature
2) Observational Diagram = $M_V$ vs. $B-V$ or Spectral Type

$\rightarrow$ Both $B-V$ and Spectral Type are functions of $T_e$

• Most stars fall along band called the main sequence

$\rightarrow$ Stars stay on main sequence for most of their lives, burning $H \rightarrow He$

$\rightarrow$ Mass determines position along the main sequence

$\rightarrow$ Different mass stars follow different tracks along this diagram. For example:

$10M_\odot$ star $\rightarrow 10^7$ years

$2M_\odot$ star $\rightarrow 10^9$ years

$1M_\odot$ star $\rightarrow 10^{10}$ years

• Estimating Lifetimes - Ref: B&M p280

To estimate the MS lifetime (i.e time to burn H cores)

$\rightarrow$ 26.7 MeV released everytime $4H \rightarrow He + \nu +$ photons

$\rightarrow$ 26.7 MeV = 25 MeV (photons) + 1.7 MeV (neutrinos)

$\rightarrow$ The difference in mass of 4H and He is

$$4m_{\text{prot,os}} - 3.97m_{\text{prot,os}} = 0.037m_{\text{prot,os}}.$$ 

Thus the efficiency of converting mass to energy with this process is

$$0.03/4 = 0.007,$$

or 0.7%. Thus,

$$E = 0.0067\Delta m_Hc^2,$$
where \( \Delta m_H = H \) mass. So,

\[
t_{ms} = \frac{(0.007\alpha M c^2)}{L},
\]

where \( \alpha \) is some fraction of the total mass of \( H \) that is converted while the star is on the main sequence, which is 10%. Thus,

\[
t_{ms} = 10^{10} \frac{(M/M_\odot)}{(L/L_\odot)}.
\]

- To estimate lifetime of HB (i.e. time to burn He cores)
  
  \( \rightarrow \) The luminosity of the HB is \( L_{HB} = 50L_\odot \)
  
  \( \rightarrow \) During a star’s life on the HB, it converts \( M(\text{He core}) = 0.45M_\odot \) into O and C.
  
  \( \rightarrow \) About 1/2 He \( \rightarrow \) O and 1/2 He \( \rightarrow \) C.
  
  \( \rightarrow \) Thus

\[
E = 7.2 \times 10^{-4} \Delta Mc^2
\]

Thus,

\[
t_{HB} = 7.2 \times 10^{-4} \times 0.45M_\odot c^2/50L_\odot = 0.1 \text{ Gyr.}
\]

- Time for massive star to go SN

  \( \rightarrow 1.4M_\odot \) of \( H \rightarrow \text{Fe} \) before exploding
  
  \( \rightarrow E = 0.0085\Delta Mc^2 \)
  
  \( \rightarrow \) If \( L = L_3 \times 10^3 L_\odot \), then

\[
t_{SG} = 0.0085 \times 1.4M_\odot c^2/1000L_3L_\odot = 0.18(L_3)^{-1} \text{ Gyr.}
\]

4. Surface Gravity

Surface Gravity, \( g \), is also a common property used to describe stars

\[
g = GM/R^2
\]
It is important for setting the pressure gradient in stellar atmosphere, which thus affects what emission/absorption lines are observed.

The pressure gradient in stars can be derived via the equation of hydrostatic equilibrium,

\[ \frac{dP}{dr} = -g \rho \rightarrow P = GM^2/R^4 \]

For a fixed \( M \), \( P \) goes up as \( R \) goes down and vice versa.

- Derivation of Hydrostatic Equilibrium:

Consider a slab of atm with density \( \rho \), thickness \( h \), and surface area \( dA \). Let \( dP \) be the pressure increment associated with \( dh \). Then the radial force on this volume element which is due to pressure differential is:

\[ F_p = PdA - (P + dP)dA = -dPdA. \]

Because the pressure actually decreases with increasing height, \( dP \) is negative. In order to balance this force, we set \( F_p \) equal to the downward pull of gravity, \( gpdAdh \), and derive

\[ \frac{dP}{dh} = -g \rho. \]

5. Luminosity Class

Luminosity is a function of \( R \), i.e., \( L = 4\pi R^2 \sigma T_e^4 \)

- Nomenclature:
  1. supergiants
  2. bright giants
  3. giants
  4. subgiants
  5. dwarfs = Main sequence
Table 1. Dwarfs - Luminosity Class V

<table>
<thead>
<tr>
<th>Spectral Type</th>
<th>Mass $M_\odot$</th>
<th>log($L_{bol}$)</th>
<th>$M_V$</th>
<th>$M/L_V$</th>
<th>Radius $R_\odot$</th>
<th>$T_e$ K</th>
</tr>
</thead>
<tbody>
<tr>
<td>03</td>
<td>120</td>
<td>6.15</td>
<td>-6.0</td>
<td>0.01</td>
<td>15</td>
<td>52500</td>
</tr>
<tr>
<td>B3</td>
<td>7.6</td>
<td>3.28</td>
<td>-1.6</td>
<td>0.03</td>
<td>4.8</td>
<td>18700</td>
</tr>
<tr>
<td>A5</td>
<td>2.0</td>
<td>1.15</td>
<td>1.9</td>
<td>0.2</td>
<td>1.7</td>
<td>8200</td>
</tr>
<tr>
<td>G0</td>
<td>1.05</td>
<td>0.18</td>
<td>4.4</td>
<td>1.1</td>
<td>1.0</td>
<td>6030</td>
</tr>
<tr>
<td>K5</td>
<td>0.7</td>
<td>-0.82</td>
<td>7.4</td>
<td>10.7</td>
<td>0.72</td>
<td>4350</td>
</tr>
<tr>
<td>M0</td>
<td>0.5</td>
<td>-1.11</td>
<td>8.8</td>
<td>30</td>
<td>0.60</td>
<td>3850</td>
</tr>
</tbody>
</table>

Adapted from Table 3.13, B&M pg. 110.

Table 2. Red Giants - Luminosity Class III

<table>
<thead>
<tr>
<th>Spectral Type</th>
<th>Mass</th>
<th>log($L_{bol}$)</th>
<th>$M_V$</th>
<th>Radius $R_\odot$</th>
<th>$T_e$ K</th>
</tr>
</thead>
<tbody>
<tr>
<td>G0</td>
<td>varies</td>
<td>1.5</td>
<td>⋯</td>
<td>6.0</td>
<td>5850</td>
</tr>
<tr>
<td>K5</td>
<td>varies</td>
<td>2.3</td>
<td>-0.2</td>
<td>25</td>
<td>3950</td>
</tr>
<tr>
<td>M0</td>
<td>varies</td>
<td>2.6</td>
<td>-0.4</td>
<td>40</td>
<td>3800</td>
</tr>
</tbody>
</table>

Adapted from Table 3.14, B&M pg. 110.
Elliptical Galaxies → old populations of stars
Spiral Galaxies (outer parts) → young+old populations of stars.

6. Remnants → condensed matter

For $M_{\text{star}} < 8M_{\odot}$ → white dwarf
  → leftover after much mass loss
  → $M_{\text{WD}} \sim 0.55 - 0.6M_{\odot}$
For $8M_{\odot} \lesssim M_{\text{star}} \lesssim 60M_{\odot}$ → neutron stars.
  → optically invisible, but visible as radio pulsars
  → $M_{\text{NS}} \sim 1.4 M_{\odot}$
For $M_{\text{star}} > 60M_{\odot}$ → Black Hole
  → optically invisible.
  → $M_{\text{BH}} > 1.4M_{\odot}$.

7. Very Low Mass Stars

For $M = 0.0001 - 0.08M_{\odot}$ → black dwarfs
  → Supported by e- degeneracy pressure.
  → Brown Dwarfs → burning De and Li, but never H.

8. Luminosity Functions

It is useful to speak of stars in a galaxy collectively.

$$dN = \Phi(M, x) dM d^3x,$$

where $dN$ is the number of stars with absolutes mag $(M + dM, M)$ within a volume $d^3x$.
$\Phi(M, x) = \Phi(M) \nu(x)$, such that,

$$dN = [\Phi(M) dM] [\nu(x) d^3x].$$
\[ \Phi(M) = \text{luminosity function - relative fraction of stars of different luminosities.} \]
\[ \nu(x) \text{ total number density of stars are point } x. \]

- From an analysis of the solar neighborhood:
  1) Most stars are intrinsically faint.
  2) Intrinsically luminous stars contribute most of the light.
  3) Most of the Mass comes from Low Luminosity Stars.

\[ 0.036 \ M_\odot \ pc^{-3}, \text{ but probably 0.039 } M_\odot \ pc^{-3} \text{ (i.e., we're missing white dwarfs).} \]
\[ \text{4) The average } M/L \sim 1M_\odot/L_\odot. \]
\[ \rightarrow \text{This is a lower limit because we're missing remnants.} \]

9. Evolution in the HR Diagram as a Function of Time

**Question:** The star formation in the solar neighborhood is constant. Will the HR Diagram of stars in the solar neighborhood look different in the future?

**Answer:** Yes

**Why?** The star formation rate in the local neighborhood is constant. So, the HR diagram is dependent on the number of stars of different masses formed (i.e., the initial mass function - see below) and the time since star formation first began.

To understand how the HR diagram will change as a function of time, consider two Populations of stars that will live for times \( \tau_1 \) and \( \tau_2 \), where \( \tau_2 >> \tau_1 \).

After time \( t << \tau_1 \), the number of stars in Population 1 and 2, \( N_1 \) and \( N_2 \) is given by,

\[ N_1 = \frac{dN_1}{dT} t \quad \text{and} \quad N_2 = \frac{dN_2}{dT} t, \]

where \( dN_1/dT \) and \( dN_2/dT \) are the star formation rates of Populations 1 and 2, respectively.

After a time \( t > \tau_1 \), but \( t < \tau_2 \), the oldest stars in Population 1 will begin to disappear. Thus, the number of Population 1 and 2 stars are,

\[ N_1 = \frac{dN_1}{dT} t - \frac{dN_1}{dT} (t - \tau_1) = \frac{dN_1}{dT} \tau_1, \]

and
\[ N_2 = \frac{dN_2}{dT} t. \]

Thus, the number of stars in Population 1 reaches an equilibrium at \( t = \tau_1 \). The number of stars in Population 2 will continue to grow until \( t = \tau_2 \).

Thus, the HR diagram of the Populations of stars will continue to evolve until \( t = \tau_2 \), where

\[ N_1 = \frac{dN_1}{dT} \tau_1 \quad \text{and} \quad N_2 = \frac{dN_2}{dT} \tau_2. \]

→ By analogy, the HR diagram of star in the solar neighborhood will continue to evolve until the first generation of white dwarfs created by the lowest mass (i.e., the longest living) stars fade from view.

10. Initial Mass Function

The IMF, \( \xi(M) = \) distribution in mass of freshly formed stars.

Consider a starburst.

\[ dN = N_0 \xi(M) dM \]

where \( dN \) is the number of stars with mass \((M, M + dM)\).

\( N_0 \) is the normalization constant with respect to mass (not according to number),

\[ \int dM M \xi(M) = M_\odot. \]

Thus, \( N_0 \) is the number of solar masses contained in the starburst.

- Determination of \( \xi \)

1) Determine \( \Phi(\text{Mag}) \) for MS (solar neighborhood, or cluster)

2) Correct \( \Phi(\text{Mag}) \) for stellar evolution effects.

→ if starburst, no correction needed

→ if SF is constant, the initial luminosity function is:

\[ \Phi_0(\text{Mag}) = \Phi(\text{Mag}) \times \frac{t}{\tau_{MS}} \]
for stars with MS life less than time since the initial stars formed.

\[ \Phi_0(\text{Mag}) = \Phi(\text{Mag}) \]

for stars with MS life longer than time since the initial stars formed.

3) Now, convert from magnitudes to mass

\[ \xi(M) = \frac{d(\text{Mag})}{dM} \Phi_0[\text{Mag}(M)]. \]

→ This can be done using models. Good for \( M_{\text{star}} > 0.5 M_\odot \).

→ Observationally – using mass determinations from binary stars.

• Bottom Line –

1) (Salpeter IMF)

\[ \xi(M) \propto M^{-2.35} \text{ (Salpeter IMF)} \]

2) (Scalo IMF) For \( M \geq 0.2 M_\odot \)

\[ \xi(M) \propto M^{-2.45} \text{ for } M > 10 M_\odot \]

\[ \xi(M) \propto M^{-3.27} \text{ for } 1 M_\odot > M > 10 M_\odot \]

\[ \xi(M) \propto M^{-1.83} \text{ for } M < 0.2 M_\odot \]

11. Analytic Solution of Main Sequence and Giant Luminosities from IMF

Ref: The Milky Way as a Galaxy, pg 275-277

Using the IMF in the form described above (simple power law), it’s possible to determine the contribution of main sequence (MS) and Giant stars to the overall luminosity of the population. Let \( \alpha = 1 + x \). Then, the IMF \( \xi(M) \) can be expressed as

\[ \xi(M)dM = CM^{-(1+x)}dM \text{ for } M_L < M < M_U. \]

Taking \( M_U >> M_L \) and \( x > 0 \), the constant \( C \) can be normalized,

\[ 1 = C \int_{M_L}^{M_U} M^{-(1+x)}dM = C \frac{M^{-x}}{x} \bigg|_{M_L}^{M_U}, \]
\[ 1 = \frac{C}{x} [M_U^{-x} - M_L^{-x}], \]

\[ C = x M_L^x. \]

Let \( M^\beta = L \), and \( t_{ms} = M^{-\gamma} \). Then,

\[ dt_{ms} = -\gamma M^{-(\gamma+1)} dM. \]

Assume that \( N_0 \) stars are created in a single burst of star formation. At time \( t \), stars with \( M > M_t = t^{-1/\gamma} \) are no longer on the main sequence.

The light from MS stars is thus,

\[ L_{ms}(t) = \int_{M_L}^{M_t} N_0 L_\xi(M) dM = N_0 C \int_{M_L}^{M_t} M^\beta M^{-(1-\gamma)} dM, \]

\[ L_{ms}(t) = N_0 x M_L^x \int_{M_L}^{M_t} M^{\beta-x-1} dM = \frac{N_0 x M_L^x}{\beta - x} [M_t^{\beta-x} - M_L^{\beta-x}], \]

\[ L_{ms}(t) = \frac{N_0 x}{\beta - x} [M_t^x M_t^{\beta-x} - M_L^x], \]

\[ L_{ms}(t) = \frac{N_0 x}{\beta - x} M_L^x M_t^{\beta-x}, \]

for \( M_L < < M_t \).

The number of giants at time \( t \) (for \( t_g < < t \)) is,

\[ N_g(t) = N_0 \xi(M_t) \left[ \frac{dM}{dt_{ms}} \right]_{M=M_t} t_g = \frac{N_0 x}{\gamma} M_L^x M_t^{\gamma-x} t_g, \]

where \( dM/dt_{ms} \) is the mass in giants generated per MS lifetime and \( t_g \) is the time since the giants in question were created (note that for post-main sequence star of a given mass \( M_t \), \( t_g = t - t_{ms} \)). In other words,

\[ \left[ \frac{dM}{dt_{ms}} \right]_{M=M_t} t_g = \left[ \frac{dM}{dt_{ms}} \right]_{M=M_t} (t - t_{ms}), \]

is simply the number of giants of mass \( M_t \) that have been created.
The total luminosity of a single burst is thus,

\[ L_{SB}(t) = L_{ms}(t) + N_g(t)L_g. \]

Setting \( x = 1.35 \) (i.e., Salpeter IMF \( \xi = M^{-2.35} \)), \( \gamma = 3 \) (\( t_{MS} = M^{-3} \)), and \( \beta = 4.9, 4.5, \) and 4.1 for \( U, B, \) and \( V \)-band, respectively (i.e, \( M^\beta = L \)), the model data can be developed.

[see Model data plotted as a function of \( U - B \) and \( B - V \)]

12. Formation of Stars

What conditions are needed to induce star formation?

**Jean Mass:** consider a spherical distribution of gas with density \( \rho \) undergoing collapse. At radius \( r \), the acceleration due to gravity is:

\[ g = -\frac{GM(r)}{r^2}. \]

The acceleration can be rewritten in the following fashion,

\[ g = \frac{d^2r}{dt^2} = v \frac{dv}{dr}, \]

where \( v \) is the velocity at radius \( r \), such that the equation of motion becomes

\[ v \frac{dv}{dr} = -\frac{GM(r)}{r^2}. \]

During the collapse, \( M(r) \) is constant. Thus,

\[ \int_{v=0}^{v} v dv = \int_{r=\infty}^{r} -\frac{GMdr}{r^2}. \]

Which can be solved for velocity \( v \),

\[ \frac{v^2}{2} = \frac{GM}{r} \rightarrow v = \left(2\frac{GM}{r}\right)^{1/2}. \]

If the gas is in **free fall collapse**, then the time for the free fall to occur is given by

\[ t_{ff} = \frac{r}{v}. \]
Substituting in the expression for mass in terms of $\rho$ and $r$,

$$M = \frac{4}{3}\pi r^3,$$

the free fall time becomes,

$$t_{ff} \sim \left(\frac{3}{8\pi G\rho}\right)^{1/2} \propto (G\rho)^{-1/2}.$$

- We will assume that the collapse is occurring isothermally. Thus the collapse will occur when $t_{ff} <$ the time taken for a sound wave to cross the cloud:

$$t_{ff} < t_{\text{sound}}.$$

The sound wave crossing time can be expressed as,

$$t_{\text{sound}} \sim \frac{r}{v_{\text{sound}}} \sim \frac{r}{(8kT/\mu m_H)^{1/2}}.$$

Thus the criterion for collapse is,

$$\left(\frac{3}{8\pi G\rho}\right)^{1/2} < \frac{r}{(8kT/\mu m_H)^{1/2}}.$$

Squaring both sides and making a substitution of $r = (3M/4\pi \rho)^{1/3}$ yields,

$$\frac{3}{8\pi G\rho} < \left(\frac{3M}{4\pi \rho}\right)^{2/3} \frac{\pi \mu m_H}{8kT}.$$

Solving for mass yields,

$$M_{\text{collapse}} \gtrsim \left(\frac{kT}{G\mu m_H}\right)^{3/2} \frac{1}{\rho^{3/2}}.$$

The Jeans Mass is the minimum mass under which collapse can occur,

$$M_J \sim \left(\frac{kT}{G\mu m_H}\right)^{3/2} \frac{1}{\rho^{3/2}}.$$

- How big is $M_J$?

For neutral, molecular gas in the interstellar medium,
\[ \mu = 2, \ T = 100 \ K, \ N = 10^6 \ \text{m}^{-3}. \]

So,

\[ \rho (\ \text{kg m}^{-3}) = (\mu m_H)N \sim 3.3 \times 10^{-21} \ \text{kg m}^{-3}, \]

Thus, \( M_J \sim 10^{34} \ \text{kg} \sim 10^4 \ M_\odot \). This mass is approximately the lower limit mass of a globular cluster, but is obviously much higher than the mass of stars. Taking more extreme values of the above parameters can lower the mass to 50–100 \( M_\odot \), but these masses are still too high (most stars have masses of \( \sim 1 \ M_\odot \)).

- The obvious solution is **fragmentation**.

  As the collapse proceeds, \( M_J \downarrow \) as \( \rho \uparrow \) because \( M_J \propto 1/\rho^{1/2} \) (provided \( T \sim \text{constant} \)).

  → Thus, the collapsing cloud could fragment into smaller clouds of \( M \sim M_J \).

  Can \( T \sim \text{constant} ? \) Yes.

  Gravitational Energy \( \rightarrow \) Thermal Energy \( \rightarrow \) Radiative Energy (\( h\nu[IR] \))

  → IR photons are an efficient way of cooling because they have a long mean free path

  \[ l \sim (N\sigma_{\text{absorption}})^{-1} \] (which will be discussed shortly).

- Summary of what can ultimately halt collapse:

  1) High \( \rho \rightarrow \) the clouds becomes opaque even to infrared photons. Cloud \( T \uparrow; \ P \uparrow; \) halt collapse.

  2) Rotation Effects

  3) Magnetic Field Effects

**Angular Momentum**

The angular momentum is expressed in terms of radius \( r \), momentum \( p \), mass \( m \) and velocity \( v \) as,

\[ \mathbf{L} = \mathbf{r} \times \mathbf{p} = \mathbf{r} \times m\mathbf{v} \]

\[ L = |\mathbf{L}| = mvr = m(r\omega)r = mr^2\omega, \]

where \( \omega \) is the angular velocity. For solid bodies, \( L = kMR^2\omega \), where \( k \sim \text{constant} \) and is dependent on the geometry of the solid.
→ Now, suppose the cloud initially has,

\[ r_i \sim 50 \text{ pc} \sim 10^{18} \text{ m} \quad \text{and } \omega_i \sim 10^{-8} \text{ rad/yr}. \]

If the cloud eventually has a radius of \( r_f \sim 100 \text{ AU} \sim 10^{13} \text{ m} \), we can use conservation of angular momentum,

\[ k M_{\text{cloud}} r_i^2 \omega_i = k M_{\text{cloud}} r_f^2 \omega_f, \]

and conclude that the angular velocity would then be \( \omega_f = 10^2 \text{ rad/yr} \), which corresponds to a period \( T = 2\pi/\omega \sim \text{few days} \).

→ This is way too fast! The cloud would literally fly apart.

→ Thus, angular momentum must be lost or converted during collapse.

• Magnetic Fields:

→ Interstellar \( B \sim 10^{-9} \text{ Tesla} \).

→ This field would be amplified during collapse. However, high \( B (\gtrsim 10^4 \text{ Gauss}) \) are not observed.

→ Thus \( B \)-field must also be lost during collapse somehow.

• Solutions:

1) Massive clouds may fragment into \( \sim 1 \text{ M}_\odot \) pieces.

2) Cloud spin angular momentum may be transported into relative angular momentum of fragments and removed by \( B \)-field.

3) \( B \)-field may reconnect and/or be trapped in stars.

• The increased angular velocity will lead to flattened disk.

In the plane of the disk, the acceleration due to gravity at a radius \( r \) is equal to

\[ g = G M_{\text{star}}/r^2. \]

Perpendicular to the disk at a radius \( r \), the acceleration at a height \( h \) above the disk is,

\[ g_\perp = g \sin \theta \sim \frac{G M_{\text{star}}}{r^2} \theta \sim \frac{G M_{\text{star}}}{r^2} \left( \frac{h}{r} \right), \]

where \( \sin \theta = h/r \), and \( \theta \sim \text{small} \).
→ If the gas in the disk is in hydrostatic equilibrium, then,

\[
\frac{dp}{dh} = -p g_{\perp} = -\rho \frac{GM_{\text{star}}}{r^3} h.
\]

From the Perfect Gas Law,

\[
\frac{dp}{dh} = \frac{kT}{\mu m_H} \frac{d\rho}{dh}.
\]

Setting the right hand sides of both equations equal to each other and solving for \( \rho \) yields,

\[
\rho = \rho(0) \exp \left[ -\left( \frac{h}{H} \right)^2 \right],
\]

where the scale height \( H \) is,

\[
H = \left( \frac{2kT r^3}{\mu m_H GM_{\text{star}}} \right)^{1/2}.
\]

Thus, the gaseous component is a flared disk.

- The dust distribution in the disk is different.
  → Not subject to hydrostatic equilibrium, just drag force \( F_{\text{drag}} \) and gravity, so it sinks to the midplane.

\[
m_{\text{dust}} \frac{d^2 h}{dt^2} = F_{\text{drag}} - m_{\text{dust}} \frac{GM_{\text{star}}}{r^3} h.
\]

→ initially, the dust grains grow in size by sticking together.

→ When enough mass has accreted in this fashion, material can be captured via self gravity.

→ Accretion leads to planet formation in the midplane of the disk.