Nearly Normal Galaxies 3: Spiral Galaxies

- Many of the properties of spiral galaxies have been previously discussed.
- Star formation in galaxies, much of which is occurring in spiral galaxies, will be discussed in the next section.
- **Present focus:** structure of spiral galaxies and dynamics of disk structure.
Review of Properties Previously Discussed

• Spiral galaxies are comprised of a bulge & disk component
• Bulge → old stars        Disk → young stars
• The disk contains a large quantity of gas & dust
• $M_B \sim -18$ to -24
• $M/L_V \sim 4$
• Disks are cold (rotationally supported) may be important to maintain spiral structure
• The rotation curves of spiral galaxies rise like a solid body in the central regions, then flatten out (i.e., $v(r) = \text{constant}$). This flattening is due to the presence of a dark matter halo.
Trends with Hubble Type
(Previously discussed)

- Total luminosity ↓ with ST (Sa → Sc)
- $M / L_B$ ↓ with ST (i.e., young stars have low $M / L_B$)
- $M$ (HI) / $M$ (total) ↑ with ST (S0 → Sm, Irr)
- $M / L_B$ ↓ as $(B – V)$ ↓ (i.e., because red stars = early type & blue stars = late type)
- Bulge / Disk ↑ with increasing Hubble Type
- Tightness of the spiral arms ↓ with increasing Hubble Type
- Degree to which the arms are resolved into stars & individual emission nebulae (HII regions) ↑ with Hubble Type
Profiles

The profiles of spiral galaxies are a combination of an Inner $r^{1/4}$ - law bulge & an outer exponential disk,

$$\log I \propto R^{1/4} \quad \text{(inner)};$$

$$I(R) = I_0 e^{-\alpha r} \quad \text{(outer)}$$

Once again, $\alpha$ is the inverse scale height

(Freeman 1970)
\[ \alpha^{-1} \downarrow \text{as S0} \rightarrow \text{Im} \]

\[ \alpha^{-1} \approx 1 - 5 \text{ kpc} \]

(Freeman 1970)
Fig. 5.—Intrinsic distance-independent blue-light luminosity scale \( B(0)_c \) for the exponential disks of thirty-six galaxies against their morphological type. Broken line at \( B(0)_c = 21.65 \) is the mean for twenty-eight galaxies. NGC numbers are shown for the other eight. \( G \) denotes an estimate for the Galaxy. \textit{Filled circles}, Type I luminosity profile; \textit{open circles}, Type II luminosity profile (see Fig. 1).

\[ B(0)_c \approx \text{constant. We will return to this soon.} \]

(Freeman 1970)
Departures from an Exponential Disk Profile

1) might be due to regions of recent Star Formation

Blue Pixels are contributing to non-exponential Disk

(Talbot, Jensen & Dufour 1979)
Departures: 2) Presence of Lens recent made by resonance (bar) destruction
Departures: 3) Bulges that are really disks

- They can be fitted by an $r^{1/4}$ profile, but have spiral structure & $(v / \sigma)^*$ consistent with rotation
- You can make a bulge by transporting gas inward & igniting a nuclear starburst.
Departures: 4) Thick Disk

- Have intermediate flattening between that of a thin disk and a bulge
- Are more diffuse than thin disk
- Have a shallow luminosity gradient parallel to the major axis
- Have a rectangular box shape in thick disk galaxies seen edge-on
Thick Disk Formation

- Intermediate Dissipation
- Dynamical Heating
- A merger which partially destroyed the thin disk through heating
Mass & Luminosity

The central mass surface density can be written as,

$$\Sigma_0 = \frac{M}{L} I_0,$$

where $I_0$ is the central surface brightness. Because $M/L$ is constant with radius, the surface brightness profile,

$$I(R) = I_0 e^{-r/r_0},$$

can be written in terms of the mass surface density as,

$$\Sigma(R) = \Sigma_0 e^{-r/r_0}.$$

For a sheet model of a disk, we can write,

$$\frac{dM(r)}{dr} = 2\pi r \Sigma.$$
Mass & Luminosity, cont’

Integrating $M(r)$ from 0 to $r$,

$$M(r) = 2\pi \int_0^r r \Sigma_0 e^{-r/r_0} dr = 2\pi \Sigma_0 r_0^2 \left[ 1 - \left(1 + \frac{r}{r_0}\right) e^{-r/r_0} \right].$$

As $r \to \infty$,

$$M(\infty) = 2\pi \Sigma_0 r_0^2.$$

Similarly,

$$L(\infty) = 2\pi I_0 r_0^2.$$
We can write,

\[ L(\infty) = 2\pi I_0 r_0^2. \]

in terms of \( M_B \),

\[ 2.5 \log I = 2.5 \log(2\pi I_0) + 2.5 \log r_0^2; \]

\[-M_B = \text{constant} + 5 \log r_0. \]

Once again, the important thing is that \( I_0 = \text{constant with Hubble Type.} \)
$M_B$ vs. $\alpha^{-1}$

$-M_{B,\text{disk}} = 16.93 + 5 \log r_0$

Fig. 7.—Absolute magnitude $M_B$ against the logarithm of the length scale $\alpha^{-1}$ (kpc). Straight line represents $[M_B, \log(\alpha^{-1})]$-relation for exponential disks with $B(0)c = 21.65$ mag per square second of arc; see eq. (22). Coding is same as for Fig. 6.

$\alpha^{-1} \downarrow$ as $M_B$ (Freeman 1970)
Total Angular Momentum

The total angular momentum of a disk can be approximated by first considering stars in circular orbits at the scale length radius $r_0$,

$$\frac{v^2}{r_0} \approx \frac{GM}{r_0^2},$$

which can be rewritten as,

$$v \approx \left( \frac{GM}{r_0} \right)^{1/2}.$$

It turns out that the total angular momentum is approximately equal to,

$$H \approx Mvr_0 = M \left( \frac{GM}{r_0} \right)^{1/2} r_0 \approx (GM^3r_0)^{1/2}.$$

The actual equation derived by Freeman (1970) for a rotating exponential disk is,

$$H = 1.109 \left( GM^3r_0 \right)^{1/2}.$$
Faber-Jackson Relation for Elliptical Galaxies

\[ L \sim \sigma^4(0) \]

Fig. 6. Correlation between central velocity dispersion \( \sigma \) and absolute magnitude \( M_B \) for elliptical galaxies and for bulges of unbarred (SA) and barred (SB) disk galaxies. The solid line is a fit to the galaxies in the middle panel; the dashed line is a fit to the ellipticals. Except for the NGC 4826 point, this figure is from Kormendy and Illingworth (1983).
Tully-Fisher Relation

I.e., the Faber-Jackson Relation for spiral galaxies. It makes use of HI rotation curves in order to trace the kinematics of spiral galaxies.

Assume stars in circular orbits,

$$v^2 \sim \frac{GM}{r_0};$$

And express the luminosity, $L$, as,

$$L \sim I_0 r_0^2.$$

Squaring the velocity equation, then making the appropriate substitutions,

$$v^4 \sim \frac{G^2 M^2}{r_0^2} \sim \frac{G^2 M^2 I_0}{L}.$$

Solving for $L$ yields,

$$L \propto \frac{v^4}{I_0 (M/L)^2} \propto v^4.$$
Determining Parameters for Tully Fisher Relation

- Determine distances to a sample of spiral galaxies using other methods
- Observe the sample of spirals in HI. From the velocity curve, determine the full width of the HI at 20% the maximum flux density
- Take out inclination & random disk motion effects in order to get \( \Delta v = W_R \),

\[
W_R = \frac{(W_{\text{FW20M}} - W_{\text{random}})}{\sin i}.
\]
Tully-Fisher Relation

Figure 7.6 Plot of absolute magnitude in $B$- and $H$- bands as a function of velocity width for galaxies with independently determined distances. [From the data published in Pierce & Tully (1992)]

\[
M_B^i = -7.48(\log W_R^i - 2.50) - 19.55 + \Delta_B \pm 0.14, \\
M_R^i = -8.23(\log W_R^i - 2.50) - 20.46 + \Delta_R \pm 0.10, \\
M_I^i = -8.72(\log W_R^i - 2.50) - 20.94 \pm 0.10, \\
M_H^i = -9.50(\log W_R^i - 2.50) - 21.67 \pm 0.08.
\]
Tully-Fisher: $L \rightarrow \Delta v^4$ at Longer $\lambda$s

I.e., because $M / L_{\lambda>0.8\mu m}$ is

1) sensitive to light from older stars
2) less sensitive to dust

FIGURE 12.7
The Tully–Fisher relation in the optical ($B$, top) and the near IR ($H$, bottom). The sample of 217 galaxies has been binned using regressions of the two variables, and morphological types are distinguished by different symbols. The lines have slopes corresponding to $\alpha = 2$ and 4 in Eq. (12.29), but at $H$ only the latter is shown. The absolute-magnitude scales are arbitrary (that is, independent of the actual Hubble constant), and distances have been calculated using a Virgocentric inflow model. (From Aaronson and Mould 1983.)
Dynamics of the Self-Gravitating Isothermal Sheet

Poisson’s Equation in cylindrical polar coordinates is,

\[
\frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial \Phi}{\partial R} \right) + \frac{1}{R^2} \frac{\partial^2 \Phi}{\partial \theta^2} + \frac{\partial^2 \Phi}{\partial z^2} = 4\pi G \rho(R, z).
\]

If axial symmetry is assumed, and the rotation curve is flat, then,

\[
\frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial \Phi}{\partial R} \right) + \frac{\partial^2 \Phi}{\partial z^2} = \frac{1}{R} \frac{\partial}{\partial R} (RF_R) + \frac{\partial^2 \Phi}{\partial z^2};
\]

\[
\frac{1}{R} \frac{\partial}{\partial R} (v_c^2) + \frac{\partial^2 \Phi}{\partial z^2} = \frac{\partial^2 \Phi}{\partial z^2} = 4\pi G \rho(R, z),
\]

where,

\[-\frac{\partial \Phi}{\partial R} = F_R = \frac{v_c^2}{R}.\]
Assuming that the isothermal sheet has an isothermal distribution function,

\[ f = f(E_z) = \frac{\rho_0}{(2\pi \sigma_z^2)^{1/2}} e^{-E_z/\sigma_z^2}, \]

where,

\[ E_z = \Phi(z) + \frac{1}{2}v_z^2 \]

(note that the disk height \(<\!\!\!<\!)\) disk scale length), the density is thus,

\[
\rho = \int_{-\infty}^{\infty} \frac{\rho_0}{(2\pi \sigma_z^2)^{1/2}} e^{-\left(\Phi + \frac{1}{2}v_z^2\right)/\sigma_z^2} dv_z = \rho_0 e^{-\Phi/\sigma_z^2}.
\]

Poisson’s Equation is thus,

\[
\frac{\partial^2 \Phi}{\partial z^2} = 4\pi G \rho_0 e^{-\Phi/\sigma_z^2}.
\]
We can express the above equation in a non-dimensional form by making the following substitutions,

\[ \phi = \Phi / \sigma_z^2 \quad \text{and} \quad \xi = z / z_0, \]

where,

\[ z_0 = \left( \frac{\sigma_z^2}{8\pi G \rho_0} \right)^{1/2}. \]

Differentiating the substituted terms,

\[ \sigma_z^2 d\phi = d\Phi \quad \text{and} \quad z_0 d\xi = dz. \]

Thus,

\[ 2 \sigma_z^2 \frac{d^2 \phi}{dz^2} = 8\pi G \rho_0 e^{-\phi}; \]

\[ 2 \frac{d^2 \phi}{dz \left( \frac{z}{\sigma_z^2 / 8\pi G \rho_0} \right)^{1/2}} = e^{-\phi}; \]

\[ 2 \frac{d^2 \phi}{d\xi^2} = e^{-\phi}. \]
From the density expression derived from the distribution function, $\phi$ can be substituted into the previous equation such that,

$$\ln \rho = \ln \rho_0 - \phi.$$ 

Thus,

$$2 \frac{d^2 \phi}{d\xi^2} = \frac{\rho}{\rho_0},$$

which has the solution,

$$\rho = \rho_0 \text{sech}^2 \left( \frac{\xi}{2} \right),$$

or,

$$\rho = \rho_0 \text{sech}^2 \left( \frac{1}{2} \frac{z}{z_0} \right).$$

From,

$$z_0 = \left( \frac{\sigma_e^2}{8\pi G \rho_0} \right)^{1/2},$$
we can solve for \( \sigma_z \),

\[
\sigma_z^2 = 8\pi z_0^2 G \rho_0.
\]

If \( \rho_0 \) (i.e., \( \rho_{z=0} \)) at any radius, \( r \), is an exponential function of \( r \), then,

\[
\sigma_z^2 \propto \rho_0 \propto e^{-r/r_0}.
\]

Because of this, many have modified the surface brightness Profile equation to take into account the isothermal sheet Model,

\[
I(r, z) = I_0 e^{-r/r_0} \text{sech}^2(z/z_0).
\]