The Galaxy Luminosity Function

Earlier in the course, we discussed the luminosity functions of stars. We now apply a similar analysis to galaxies, both in the field and in clusters.

Consider a sample of galaxies $S$. We can define the following quantities:

- $n_s(L)$ → Number of galaxies in $S$ per unit luminosity.
- $n_s(L)\,dL$ → Number of galaxies in $S$ with luminosities between $L$ and $L + dL$
- $\phi_s(L)$ → luminosity function such that,

\[ \phi_s(L) = \frac{n_s(L)}{V_s(L)}. \]

Thus, $\phi_s(L)$ is the number of galaxies in $S$ per unit luminosity per unit volume.

On small scales, inhomogeneity is important and $\phi$ depends on $S$. If the universe is homogeneous on large scales,

\[ \phi_S(L) \to \phi(L) \quad \text{as} \quad V_s \to \infty. \]

$\phi(L)$ is referred to as the universal luminosity function.

- How do $n_s$ and $\phi_s$ differ? In a cluster, $V_S$ is the same for all $L$, so $n_s$ and $\phi_s$ have the same shape. However, if the sample is apparent-magnitude limited, then $V_s \downarrow$ as $L \downarrow$.

[See log $n_s$, log $\phi_s$ vs. log $L$ Figures]

The Schechter Function is typically used to characterize galaxy luminosity functions, and it has the form,

\[ \phi(L) = \left( \frac{\phi^*}{L^*} \right) \left( \frac{L}{L^*} \right)^\alpha e^{-\left(\frac{L}{L^*}\right)}, \]

where $\phi^*$ is the normalization density, $L^*$ is a characteristic luminosity corresponding to $M_B = -20.6$, and $\alpha$ is the power-law slope at low $L$.

This function can also be expressed in terms of magnitudes by making the substitutions,

\[ \phi(M)\,dM = \phi(L)\,d(-L); \]

and

\[ M - M^* = -2.5 \log \left( \frac{L}{L^*} \right). \]
to yield,

$$\phi(M) = \frac{\ln 10}{2.5} \phi^* 10^{0.4(\alpha+1)(M-M^*)} \exp \left[ -10^{0.4(M-M^*)} \right].$$

Typical values derived from $B$-band measurements are,

$$\phi^* = (1.6 \pm 0.3) \times 10^{-2} h^{-3} \text{Mpc}^{-3},$$

$$M_B^* = -19.7 \pm 0.1 + 5 \log 5,$$

$$\alpha = -1.07 \pm 0.07, \text{ and}$$

$$L_B^* = (1.2 \pm 0.1) h^{-2} \times 10^{10} \text{L}_\odot.$$

Because $B$-band measurement are sensitive to the affects of dust and potential irregularities introduced by star formation, a determination of the luminosity function was also done at $K$. The following quantities were determined:

$$\phi^* = (1.6 \pm 0.2) \times 10^{-2} h^{-3} \text{Mpc}^{-3},$$

$$M_K^* = -23.1 \pm 0.2 + 5 \log 5, \text{ and}$$

$$\alpha = -0.9 \pm 0.2.$$

Note the similarity in $\phi^*$ and $\alpha$ as measured in both bands.

1. Properties

- The number density of galaxies whose luminosities exceed $L$ is,

$$\frac{1}{L} \int_{L}^{\infty} \phi(L) dL = \phi^* \int_{(L/L^*)}^{\infty} \left( \frac{L}{L^*} \right)^{\alpha} e^{-L/L^*} d \left( \frac{L}{L^*} \right) = \phi^* \Gamma (\alpha + 1, L/L^*),$$

diverges for $\alpha < -1$ as $L/L^* \to 0$.

- The luminosity density of galaxies whose luminosities exceed $L$ is,

$$\int_{L}^{\infty} L \phi(L) dL = \phi^* L^* \int_{(L/L^*)}^{\infty} \left( \frac{L}{L^*} \right)^{\alpha+1} e^{-L/L^*} d \left( \frac{L}{L^*} \right) = \phi^* L^* \Gamma (\alpha + 2, L/L^*),$$

which converges for $\alpha > -2$. In other words, the Schechter function diverges by number density, but not by luminosity density. For $\alpha = -1$, the total luminosity density is

$$= \phi^* L^* \Gamma (2 + \alpha) = \phi^* L^* = 1 \times 10^8 h L_B(\odot) \text{Mpc}^{-3}.$$
Half of this luminosity density is contributed by galaxies with $L/L^* > 1/2$.

Though the number density diverges, we can determine the number density of galaxies in units of Milky Ways,

$$N_{\text{gal}} = \frac{1 \times 10^8 \text{L}_\odot \text{Mpc}^{-3}}{1.7 \times 10^{10} \text{L}_\odot} = 0.006 \text{Mpc}^{-3}.$$  

I.e., if the universe were comprised only of Milky Ways and the luminosity density was $1 \times 10^8 \text{L}_\odot \text{Mpc}^{-3}$, there would be 0.006 galaxies per Mpc$^3$.

For an apparent magnitude limited sample,

- $r$ to which an $L$ galaxy can be seen $\propto L^{1/2}$.

- $V$ to which an $L$ galaxy can be seen $\propto L^{3/2}$.

Thus,

$$n(L) \propto \phi(L)L^{3/2} \propto \left( \frac{L}{L^*} \right)^{\alpha+3/2} e^{-L/L^*}.$$  

For an $\alpha = -1.25$, which is the value for rich clusters, the above function peaks at $\sim 0.25L^*$, and the median galaxy has $L \sim L^*$.

- Three more points to note:

  1: $\phi(L)$ is best determined near $L^*$.

     → Few galaxies have $L >> L^*$ because they are rare.

     → Few galaxies have $L << L^*$ because they are too faint to see.

  2: M31 ($M_B = -20.3$) is a 0.5$L^*$ galaxy, and the combined local group $\sim 1L^*$.

  3: cD galaxies, which are $5 - 10L^*$, do not fit into the Schechter function scheme.

2. Luminosity Function as a Function of Hubble Type

[See plots of Luminosity Function Shown as a Function of Hubble Type.]

It turns out that when luminosity functions of galaxies of specific Hubble Types are made, only the faintest galaxies have Schechter luminosity functions. The luminosity functions of dwarf ellipticals and Irr's have the following parameters,

$$M_B^{*}(\text{Irr}) = -15 + 5 \log h \quad \text{and} \quad \alpha(\text{Irr}) = -0.3,$$

$$M_B^{*}(\text{dE}) = -16 + 5 \log h \quad \text{and} \quad \alpha(\text{dE}) = -1.3.$$
All the other Hubble Types have gaussian functions with the following parameters,

\[ M_B(Sa - Sc) = -16.8 \text{ and } \sigma_B = -0.3; \]

\[ M_B(SO) = -17.5 \text{ and } \sigma_B = 1.1. \]

The elliptical galaxy function is a little more complex,

\[ \Phi_E(M_B) \propto \exp \left( \frac{-[M_B^E - M_B]^2}{2\sigma_B^E(M_B)} \right); \]

where \( M_B^E = -16.9 + 5 \log h \) and \( \sigma_B^E = 2.2 \) for \( M_B < M_B^E \) and 1.3 for \( M_B > M_B^E \).

3. X-ray Emitting Gas in Clusters

One major property of rich clusters briefly mentioned in the discussion of hot gas in elliptical galaxies (see the Dark Matter lecture) is the presence of hot X-ray gas. The mass of this gas in rich clusters is in many cases equivalent to the mass of stars in the cluster galaxies. Indeed, it is through the detection of X-ray emission that many distant, massive clusters have been located.

The X-ray gas has temperatures on the order of \( 10^7-8 \) K, and thus velocity dispersions of,

\[ \sigma \sim \left( \frac{kT}{\mu m_H} \right)^{1/2} \sim 1000 - 3000 \text{ km s}^{-1}. \]

The luminosity of the X-ray gas is given by,

\[ L_X \propto n_e^2 T^{1/2} R^3, \]

where \( n_e \) is the electron number density and \( R \) is the radius of the spherical distribution of X-ray gas. Note that given the high temperatures (and thus velocity dispersions) of X-ray gas, only massive clusters have deep enough potential wells to retain such gas.

4. Cluster vs. Field

As might be expected, the relative fraction of different Hubble types changes with the galaxy density of its environment.

[See LF of field galaxies and the Virgo cluster from Binggeli, Sandage, & Tammann 1988, ARAA, 26, 509]

To summarize the major properties:
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• The fraction of Ellipticals and S0 increases with increasing clustering.
• The fraction of dE increases with increasing clustering.
• The merger fraction is effectively zero in dense clusters. This is because the velocity dispersions of clusters are extremely high (> 1000 km s⁻¹), which is in excess of the escape velocities of any galaxies passing nearby. As a result, the galaxies simply fly passed each other.
• Note also that Dwarf galaxies are associated with large galaxies. Thus it is not the case that bright galaxies formed in dense environments and faint galaxies formed in less dense environments.

[See plot of galaxy distribution by Binggeli, Tarenghi, & Sandage 1990, A&A, 228, 42]

• The fraction of spirals and irregular galaxies increases with increasing radii from the cluster center. Also, the fraction spirals increases with decreasing projected galaxy density. Which one of these properties is more fundamental is not clear, but the implication is that the cold disk component of these galaxies is disrupted by the tidal forces of nearby galaxies or pressure forces induced by hot X-ray gas in the centers of clusters.

[See plot of fraction of galaxy population vs. radius and density]

5. Low Surface Brightness Galaxies

Clearly, when forming luminosity functions, we are biased towards the nearest and/or the brightest galaxies. Thus, an obvious question is whether or not a substantial fraction of galaxies (and thus mass) is being missed.

An example of such a galaxy was found by accident in the late 1980’s by Bothun, Impey, Malin, & Mould (1987, AJ, 94, 23). The galaxy, named Malin 1 has an incredibly low central and disk surface brightness; the disk surface brightness is only 2% the brightness of the night sky.

[See two pages summarizing Malin 1’s properties]

While the disk is faint, it is incredibly large; its scale length is ~ 50 kpc. It also contains 10¹¹ M☉ of HI gas, significantly more than any other galaxy.

Malin 1 also has a Seyfert nucleus, but is otherwise quiescent. Thus, although it contains a rich ISM, the surface densities are simply too low to have star formation via disk instabilities (see the previous discussion of disk instabilities due to rotation in the Star Formation lecture).