Population Synthesis Models: Color-Color Diagram
Components of Galaxies – Dust

Where does Dust come from?

• Mass Loss From Evolving Low Mass Stars

• Supernovae
Evidence for Dust

- Abundance of Interstellar gas measured along the line of sight to near stars – not typical of Sun

- Metals missing from interstellar gas are capable of forming solids which are heat stable & resistant
Creation of Dust

• Evidence - Missing metals in interstellar gas

• Gas moves farther away from stars, cools, & condenses out of gas

• Examples:
  1) Si & O → Mg + Fe → Silicates
  2) Fe → Fe particles
  3) Si + C → Silicon carbide, graphite, metal oxides
Effect on Radiation

• Scattering (e.g. Nebulae have same spectrum as central star)

• Absorption (of UV light from Stars/AGN)
  \[ \rightarrow \text{Generation of far-infrared photons} \]
  \[ \rightarrow \text{Typical Dust temperatures } T = 10 - 60 \text{ K} \]
Absorption of Radiation from a Star by Dust

Consider a dust grain of diameter \( a = 0.1 \, \mu m \) at a distance \( r \) from a star of luminosity \( L_* \).

The flux received by the grain is

\[
\left( \frac{L_*}{4\pi r^2} \right) \pi a^2 Q_{\text{in}},
\]

where \( Q_{\text{in}} \) is the absorption efficiency. The dust grains will radiate away

\[
4\pi a^2 (\sigma T^4_{\text{dust}}) Q_{\text{out}},
\]

where \( Q_{\text{out}} \) is the emission efficiency.
Thus,

\[ T_{\text{dust}} = \left( \frac{L_*}{16\pi r^2 \sigma} \frac{Q_{\text{in}}}{Q_{\text{out}}} \right)^{1/4} \]

So, if \( L_* = 2 \times 10^{28} \text{ W} = 100 \text{ L}_\text{solar} \) and \( r = 10^{14} \text{ m} = 700 \text{ AU} \), then,

\[ T_{\text{dust}} = 30 \text{ K} \left( \frac{Q_{\text{in}}}{Q_{\text{out}}} \right)^{1/4} \]

where \( \sigma = 5.7 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4} \). From Wien’s Law, a dust grain emitting thermal radiation at this temperature emits the peak of its flux at a \( \lambda \) of

\[ \lambda_{\text{peak}} = \frac{0.3 \text{ cm K}^{-1}}{T(\text{K})} = \frac{0.3 \text{ cm K}^{-1}}{30 \text{ K}} \sim 100 \mu\text{m}. \]
Opacity

A beam of radiation with intensity $I$ traverses a slab of dust of thickness $dz$.

The amount of radiation removed from the beam is

$$dI = -I \kappa dz = -IN_{dust}(z)\sigma dz = -Id\tau,$$

where $\kappa$ is the absorption coefficient, $N_{dust}$ is the density of dust grains, $\sigma$ is the absorption cross section, and $\tau$ is the opacity.
At $\tau = 1$, a photon has traveled one mean free path, $\ell$

$$\ell = (N_{\text{dust}} \sigma)^{-1}$$

Integrating $dI = -I d\tau$ & solving for intensity yields,

$$I = I_0 e^{-\tau},$$

where $I_0$ is the intensity prior to crossing the dust slab.

In terms of opacity,

$$\tau = -2.3026 \log \frac{I}{I_0}.$$
Extinction

In terms of magnitudes of extinction at some wavelength $X$, $A_X$,$$
A_X = [m(X) - m_0(X)] = -2.5 \log \frac{I(X)}{I_0(X)} = 1.086 \tau.
$$

where $m_0$ is the magnitude in the absence of extinction, $m$ is the extinguished magnitude.

If the spectral type & luminosity of a star is known, as well as the distance $d$ to the star, then the extinction can be determined

\[ m_X - M_X = A_X + 5 \log(d) - 5, \]

where $m_X$ is the observed extinguished magnitude & $M_X$ is the absolute magnitude of the star.
Color Excess

In terms of color excess as measured between wavelengths $X$ & $Y$, $E(X - Y)$,

\[ E(X - Y) = [m(X) - m(Y)] - [m_0(X) - m_0(Y)] = A_X - A_Y. \]

I.e., color excess is the extinction between two wavelengths.
Extinction Curve in Terms of $E(B-V)$ vs. $\lambda^{-1}$

- Note that the extinction law for the Galaxy differs from that of the LMC & SMC
- Typically, the extinction is given in terms of all 3 extinction laws.
$A_\lambda$ vs. $\lambda$

- Note that $A_\lambda \approx \lambda^{-1}$ at long wavelengths

Table 3.21 The standard interstellar extinction law

<table>
<thead>
<tr>
<th>Band X</th>
<th>$E(X-V)/E(B-V)$</th>
<th>$A_X/A_V$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U$</td>
<td>1.64</td>
<td>1.531</td>
</tr>
<tr>
<td>$B$</td>
<td>1.00</td>
<td>1.324</td>
</tr>
<tr>
<td>$V$</td>
<td>0.00</td>
<td>1.000</td>
</tr>
<tr>
<td>$R$</td>
<td>-0.78</td>
<td>0.748</td>
</tr>
<tr>
<td>$I$</td>
<td>-1.60</td>
<td>0.482</td>
</tr>
<tr>
<td>$J$</td>
<td>-2.22</td>
<td>0.282</td>
</tr>
<tr>
<td>$H$</td>
<td>-2.55</td>
<td>0.175</td>
</tr>
<tr>
<td>$K$</td>
<td>-2.74</td>
<td>0.112</td>
</tr>
<tr>
<td>$L$</td>
<td>-2.91</td>
<td>0.058</td>
</tr>
<tr>
<td>$M$</td>
<td>-3.02</td>
<td>0.023</td>
</tr>
<tr>
<td>$N$</td>
<td>-2.93</td>
<td>0.052</td>
</tr>
</tbody>
</table>

SOURCE: From data published in Rieke & Lebofsky (1985)
N(H) vs. E(B – V)

$E(B - V) = \frac{N_H}{5.8 \times 10^{25} \text{m}^{-2}}$, where $N_H$ is the number density of hydrogen gas in molecular & atomic form.
Example 1

A star in a cluster located 25 pc from the Earth is observed. The spectral type of the star is known to be O8 (i.e., $M_V = -4.9$)

• If the apparent $V$ magnitude is measured to be $m_V = 5$, what is the optical extinction to this star.

$$A_V = m_V - M_V - 5 \log_{10} 25 + 5 = 7.9 \text{ mags of extinction at } V.$$  

• What is the extinction at $K$?

From Table 3.21,

$$\frac{A_K}{A_V} = 0.112.$$
• What is the opacity at $V$ & $K$?

We know that $A_X = 1.086 \: \tau_X$. Thus,

$$\tau_V = \frac{A_V}{1.086} = 7.27.$$

$$\tau_K = \frac{A_K}{1.086} = 0.81.$$

• Note that
  1) starlight traverses a path that is optically thick at $V$
  2) and optically thin at $K$

• Thus the infrared wavelength range is important because **stars form in molecular clouds.**
• In terms of the measured intensity vs. the intensity in the absence of dust, how much has the intensity been extinguished by at V & K?

\[
\left( \frac{I}{I_0} \right)_V = e^{-\tau_V} = 7 \times 10^{-4}.
\]

\[
\left( \frac{I}{I_0} \right)_K = e^{-\tau_K} = 0.44.
\]

• What is the extinction as measured at 60 µm?

Table 3.21 doesn’t list the extinction at 60 µm. But we know that \( A_x \sim \lambda_x^{-1} \) at long wavelengths.

\[
A_{60\mu m} = A_K \left( \frac{\lambda_K}{\lambda_{60\mu m}} \right) = 0.88 \left( \frac{2.2}{60} \right) = 0.0323.
\]

The corresponding values of \( \tau_{60\mu m} \) & \( (I/I_0)_{60\mu m} \) are 0.030 & 0.97.
• What is the color excess \( E(B - V) \)?

\[
E(B - V) = A_B - A_V = (1.324A_V) - A_V = 0.324A_V = 2.6,
\]

where the relation \( A_B = 1.324 \, A_V \) is taken from Table 3.21.

• Note that
1) the factor 0.324 is often referred to as \( 1/R \), where \( R \) is the slope of the extinction curve near \( E(B - V) \).
2) \( R \) has a lot of scatter, but \( R \approx 3.1 \) is still commonly used.

• Given the above \( E(B - V) = 2.6 \), what is the column density of hydrogen along the line of sight to the star?

\[
N(H_{tot}) = 5.8 \times 10^{25} \, E(B - V) \, m^{-2} = 1.5 \times 10^{26} \, m^{-2}.
\]
Example 2 – How Extreme can Extinction get?

• Recent X-ray observations of the luminous infrared galaxy NGC 6240 were used to calculate a column density of $N(H) = 2 \times 10^{28} \text{ m}^{-2}$.

• What is the color excess $E(B - V)$ & the optical extinction along the line of sight to the X-ray emitting source?

\[ E(B - V) \sim \frac{N(H)}{5.8 \times 10^{25} \text{ m}^{-2}} = 345 \text{ mags}, \]

and

\[ A_V \sim 3.1 E(B - V) = 1070 \text{ mags of optical extinction!} \]

• What is the extinction at 60 µm?

\[ A_{60\mu m} \sim A_V \left( \frac{0.55\mu m}{60\mu m} \right) = 9.8 \text{ mags of extinction at 60µm}. \]
Calculating Extinction Using Hydrogen Recombination Lines

- Extinction via stellar type has little use for distant objects
- Solution: Hydrogen recombination lines – intrinsic line ratios are known
- This technique can only be done for galaxies with strong emission lines

Starburst: OB stars

AGN: accretion disks
Energy Level Diagram – Useful Lines

• Ly\( \alpha \) at 0.1216 \( \mu \)m (n = 2 \( \rightarrow \) 1)
• H\( \beta \) at 0.4861 \( \mu \)m (n = 4 \( \rightarrow \) 2)
• H\( \alpha \) at 0.6563 \( \mu \)m (n = 3 \( \rightarrow \) 2)
Energy Level Diagram – Relative Transition Rates

3 _____ _____ _____

2 _____ _____

Hα photons

Lyα photons

Two continuum photons

l.d.l. → ⅔ Lyα + ⅓ • 2(continuum γ)

h.d.l. → 1 Lyα + 0 (continuum γ)

• For high density limit (h.d.l.), N_e > 10^{11} cm⁻³. Collisions are important.
The intrinsic line ratios can be determined if the relative transition rates are known. For example, if we consider Ly\(\alpha\) & H\(\alpha\),

\[
\frac{I_{Ly\alpha}}{I_{H\alpha}} = \frac{I_{Ly\alpha}}{I_{H\alpha}} = K \frac{\alpha_B h \nu_{Ly\alpha}}{\alpha_{H\alpha} h \nu_{H\alpha}}.
\]

where

- \(K = \) number of Ly\(\alpha\) photons produced per H\(\alpha\) photon
- \(\alpha_B = \) recombination rate summed over all levels above ground level (cm\(^3\) s\(^{-1}\))
- \(\alpha_{H\alpha} = \) effective recombination coefficient for H\(\alpha\)

Intrinsic ratios typically used are,

\[
\frac{Ly\alpha}{H\alpha} \sim 8.1 \text{ (Starburst) or } \sim 16 \text{ (AGN)},
\]

\[
\frac{H\alpha}{H\beta} \sim 2.85 \text{ (Starburst) or } \sim 3.1 \text{ (AGN)}
\]
Color Excess Calculations using Recombination Lines

To calculate the color excess $E(B - V)$, the intensities of two recombination lines must be measured, then an extinction curve must be adopted.

From the definition of magnitude & color excess,

$$E(\lambda_2 - \lambda_1) = 2.5 \left( \log \left( \frac{I_2}{I_1} \right)_{\text{intrinsic}} - \log \left( \frac{I_2}{I_1} \right)_{\text{measured}} \right)$$

From the extinction curve, we can solve for the quantity,

$$\frac{E(\lambda_2 - \lambda_1)}{E(B - V)} = \frac{E(\lambda_2 - V)}{E(B - V)} - \frac{E(\lambda_2 - V)}{E(B - V)}.$$
Example 3

Ly$\alpha$ & H$\alpha$ are measured from a redshift $z \sim 2.2$ radio galaxy TX 0200+015. The ratio of these lines are determined to be Ly$\alpha$ / H$\alpha$ $\sim 1.7$

- What is the color excess $E(B - V)$ along the line of sight to the line-emitting gas?
  
  From the extinction curve for the galaxy, we can determine that

\[
\frac{E(\text{Ly} \alpha - V)}{E(B - V)} = \frac{E(0.1216\mu m - V)}{E(B - V)} = 6.95 \quad \text{and} \quad \frac{E(\text{H} \alpha - V)}{E(B - V)} = \frac{E(0.6563\mu m - V)}{E(B - V)} = -0.83.
\]

Thus,

\[
\frac{E(\text{Ly} \alpha - \text{H} \alpha)}{E(B - V)} = \frac{E(0.1216\mu m - V)}{E(B - V)} - \frac{E(0.6563\mu m - V)}{E(B - V)} = 7.78 \quad \text{(Milky Way)}.
\]

$E(\text{Ly} \alpha - \text{H} \alpha) / E(B - V) \sim 11.22$ (LMC & SMC extinction curves)
Example 3, cont.

Now,

\[ E(\text{Ly}\alpha - \text{H}\alpha) = 2.5 \left[ \log \left( \frac{I_{\text{Ly}\alpha}}{I_{\text{H}\alpha}} \right)_{\text{intrinsic}} - \log \left( \frac{I_{\text{Ly}\alpha}}{I_{\text{H}\alpha}} \right)_{\text{measured}} \right]. \]

Thus,

\[ E(\text{Ly}\alpha - \text{H}\alpha) = 2.5[\log(16) - \log(1.7)] = 2.43. \]

Taking the two expressions for $E(\text{Ly}\alpha - \text{H}\alpha)$ & setting them equal to each other, & solving for $E(B - V)$, we get,

\[ E(B - V) = 0.31 \ (\text{MW}), \ \sim 0.21 \ (\text{LMC}), \ \text{and} \ \sim 0.14 \ (\text{SMC}). \]