Notes on logarithms.

Suppose that a quantity $x$ is expressed as a power of some quantity $a$. We may write

$$x = a^y$$

where $y$ is the power. The number $a$ is called the base number. The logarithm of $x$ with respect to the base $a$ is equal to the exponent to which the base must be raised in order to satisfy the expression $x = a^y$. We write this as

$$y = \log_a x.$$ 

This expression says that $y$ is the power to which you must raise $a$ to get $x$. We will deal with “base 10” or common logs, meaning that $a = 10$. Examples:

\[
\begin{align*}
\log_{10} 1 &= 0.0 & \text{which says that } 1 &= 10^0 \\
\log_{10} 2 &= 0.30103 & \text{which says that } 2 &= 10^{0.30103} \\
\log_{10} 8 &= 0.90309 \\
\log_{10} 10 &= 1.0 \\
\log_{10} 20 &= 1.30103 \\
\log_{10} 80 &= 1.90309 \\
\log_{10} 100 &= 2.0
\end{align*}
\]

We also can write the antilog

$$x = \text{antilog}_a y,$$

which means that $x$ is the number you get when you raise $a$ to the power $y$, or

$$x = a^y.$$ 

Note that this is called the antilog because it acts to undo the log. If $u = \log_a v$, then $v = \text{antilog}_a u$. Example: $\log_{10} 52 = 1.716$, so $\text{antilog}_{10} 1.716 = 10^{1.716} = 52$

Properties of logs:

\[
\begin{align*}
\log(ab) &= \log(a) + \log(b) \\
\log(a/b) &= \log(a) - \log(b) \\
\log(a^n) &= n \log(a) \\
\log \left( \frac{1}{a} \right) &= -\log(a)
\end{align*}
\]